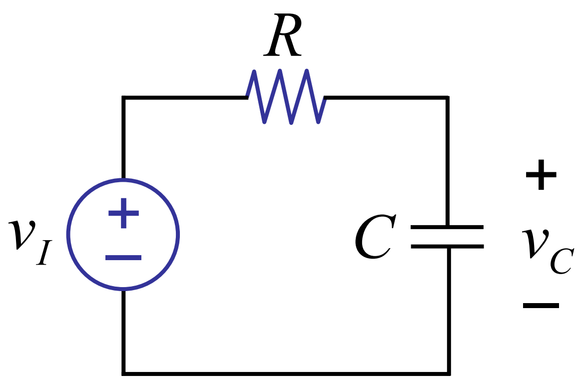
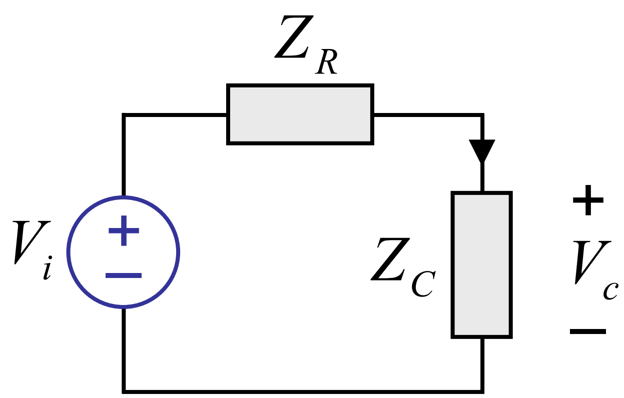
Filters

Review



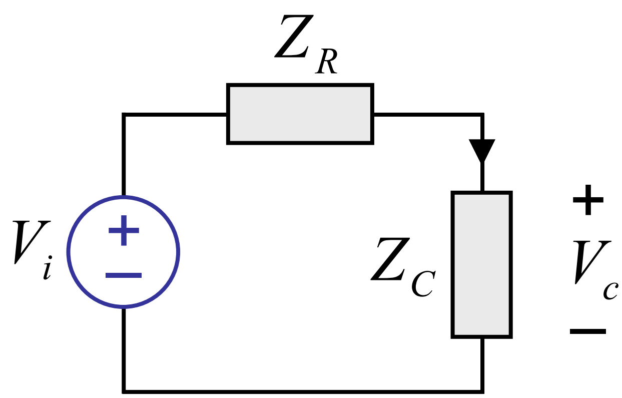


$$V_c = \frac{Z_C}{Z_C + Z_R} \cdot V_i$$

$$\frac{V_c}{V_i} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$

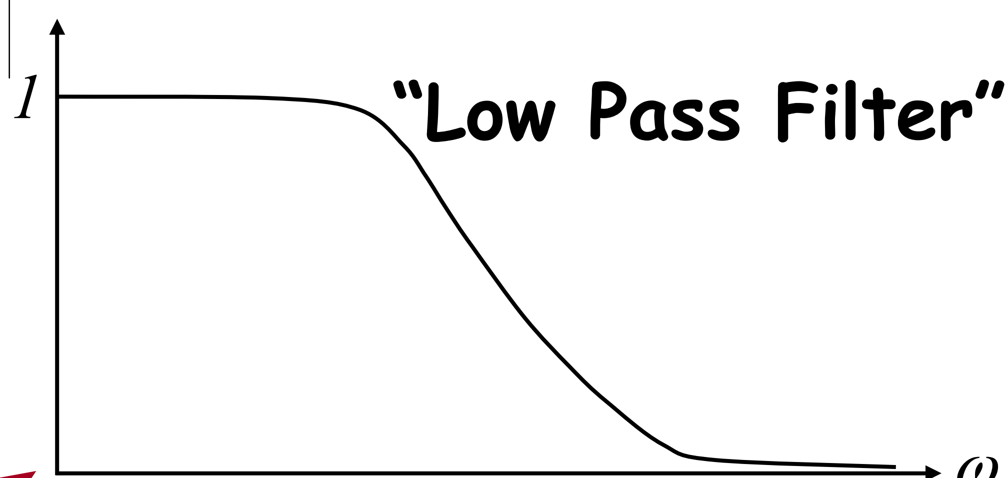
Reading: Section 14.5, 14.6, 15.3 from A & L.

A Filter



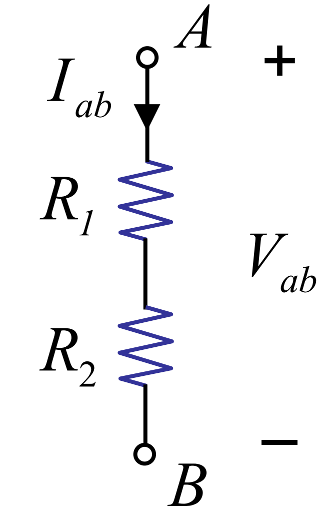
$$V_c = \frac{Z_C}{Z_C + Z_R} \cdot V_i = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \left|\frac{V_c}{V_i}\right|$$

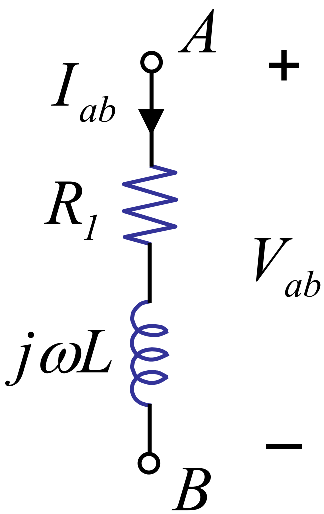


Quick Review of Impedances-

Just as



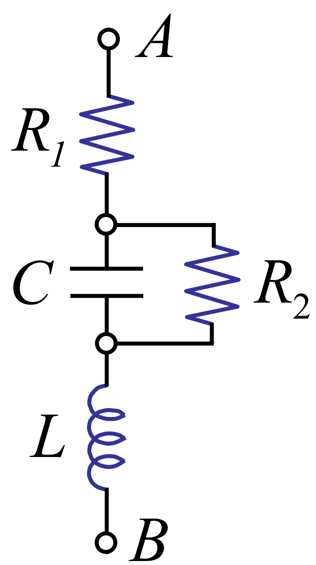
$$R_{AB} = \frac{V_{ab}}{I_{ab}} = R_1 + R_2$$



$$Z_{AB} = \frac{V_{ab}}{I_{ab}} = R_1 + j\omega L$$

Quick Review of Impedances

Similarly

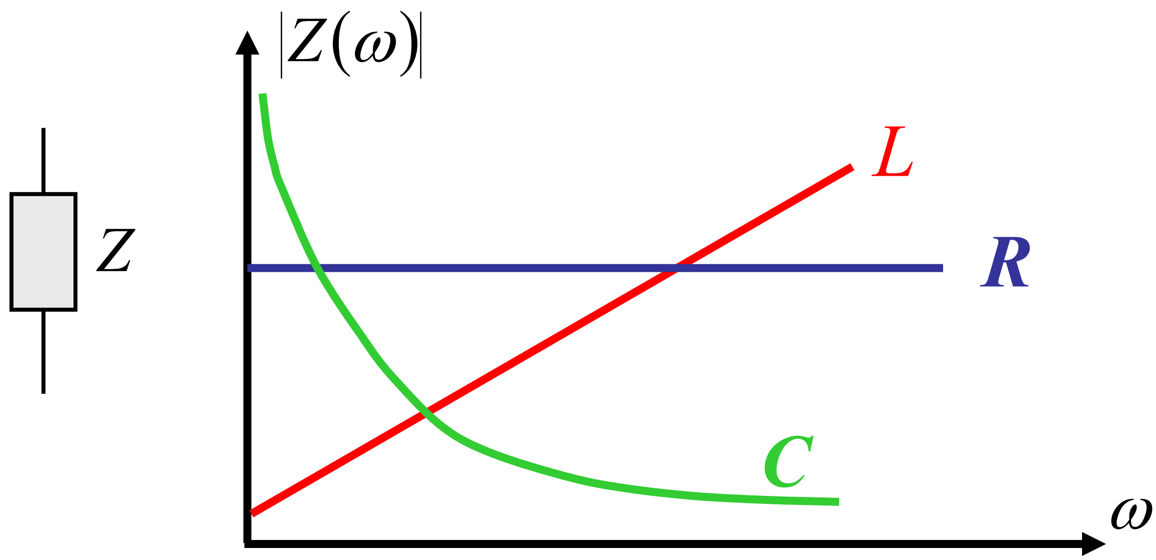


$$Z_{AB} = R_1 + Z_C \,||\, R_2 + Z_L$$

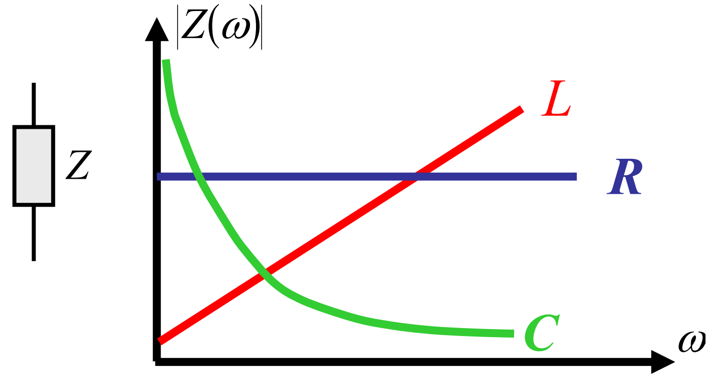
$$= R_1 + \frac{Z_C R_2}{Z_C + R_2} + Z_L$$

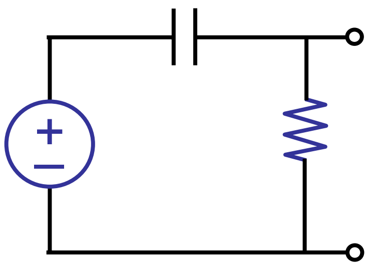
$$= R_1 + \frac{R_2}{1 + j\omega C R_2} + j\omega L$$

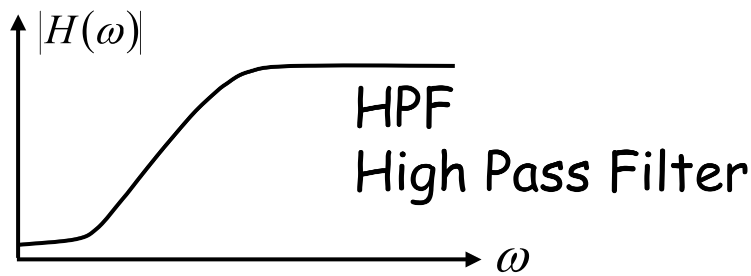
We can build other filters by combining impedances

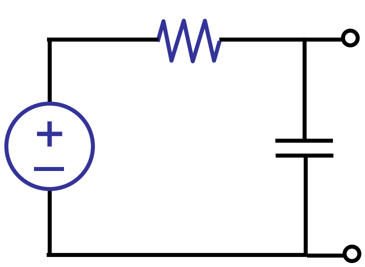


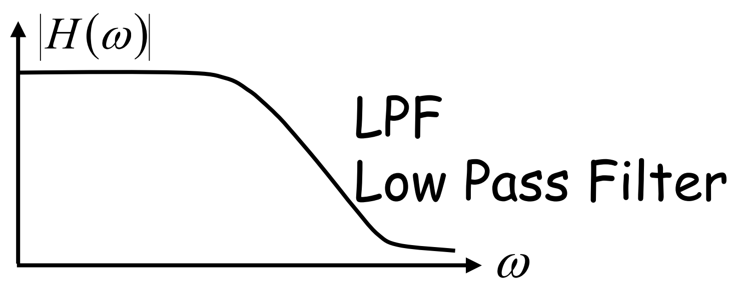
We can build other filters by combining impedances

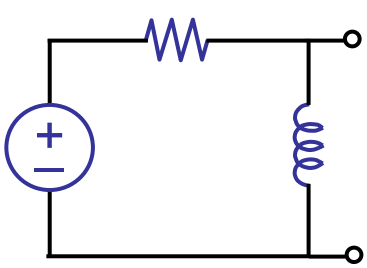


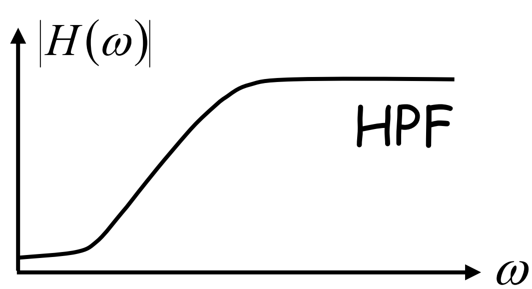




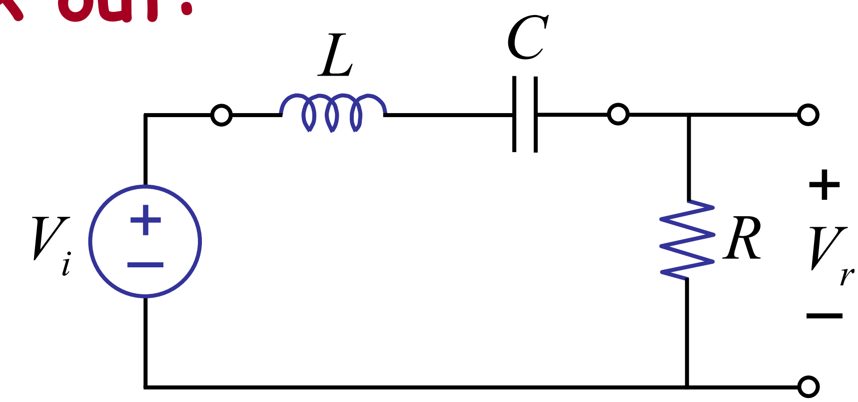




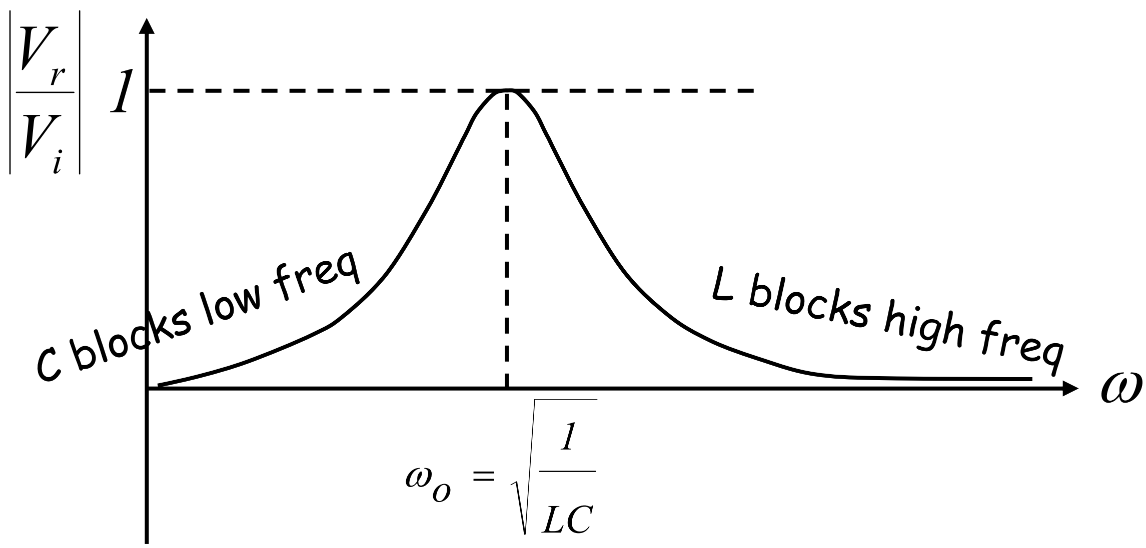




Check out:



Intuitively:



$$\frac{V_r}{V_i} = \frac{R}{j\omega L + \frac{1}{j\omega C} + R}$$

$$= \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}$$

$$\left|\frac{V_r}{V_i}\right| = \frac{\omega RC}{\sqrt{\left(1 - \omega^2 LC\right)^2 + (\omega RC)^2}}$$

At resonance,

$\omega = \omega_o$

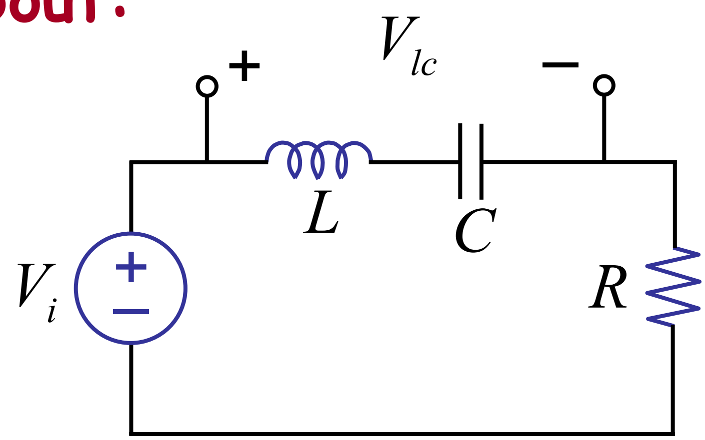
and

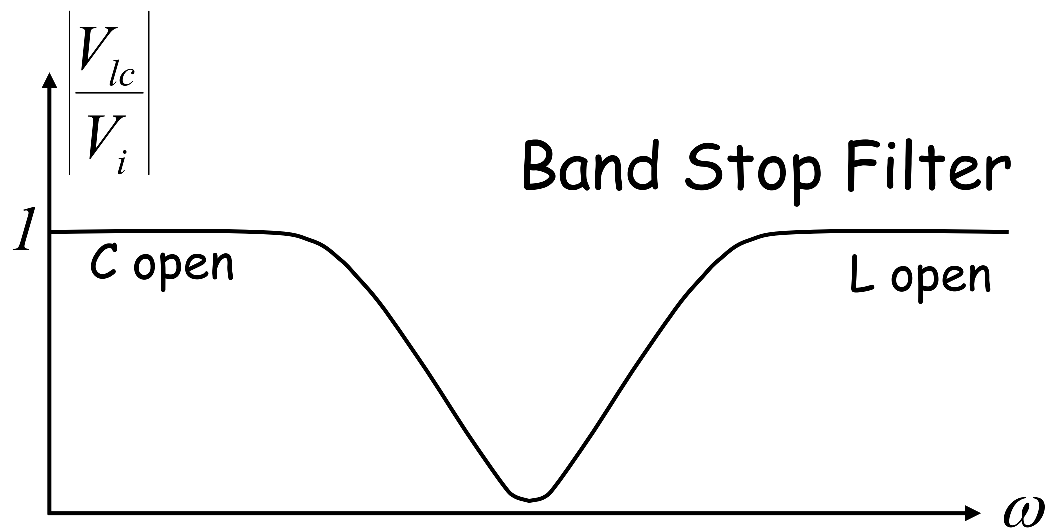
$Z_L + Z_C = 0$,

so V_i sees only R!

More later…

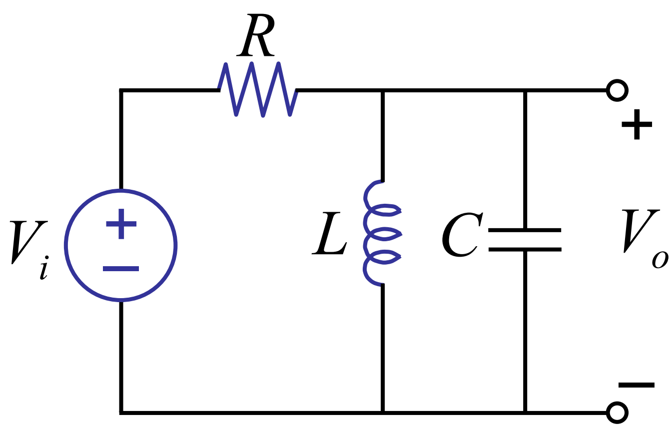
What about:

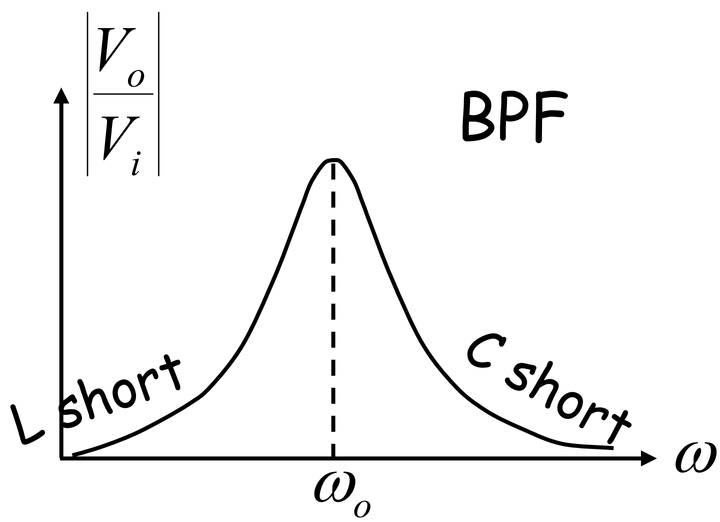




Check out V_l and V_c in the lab.

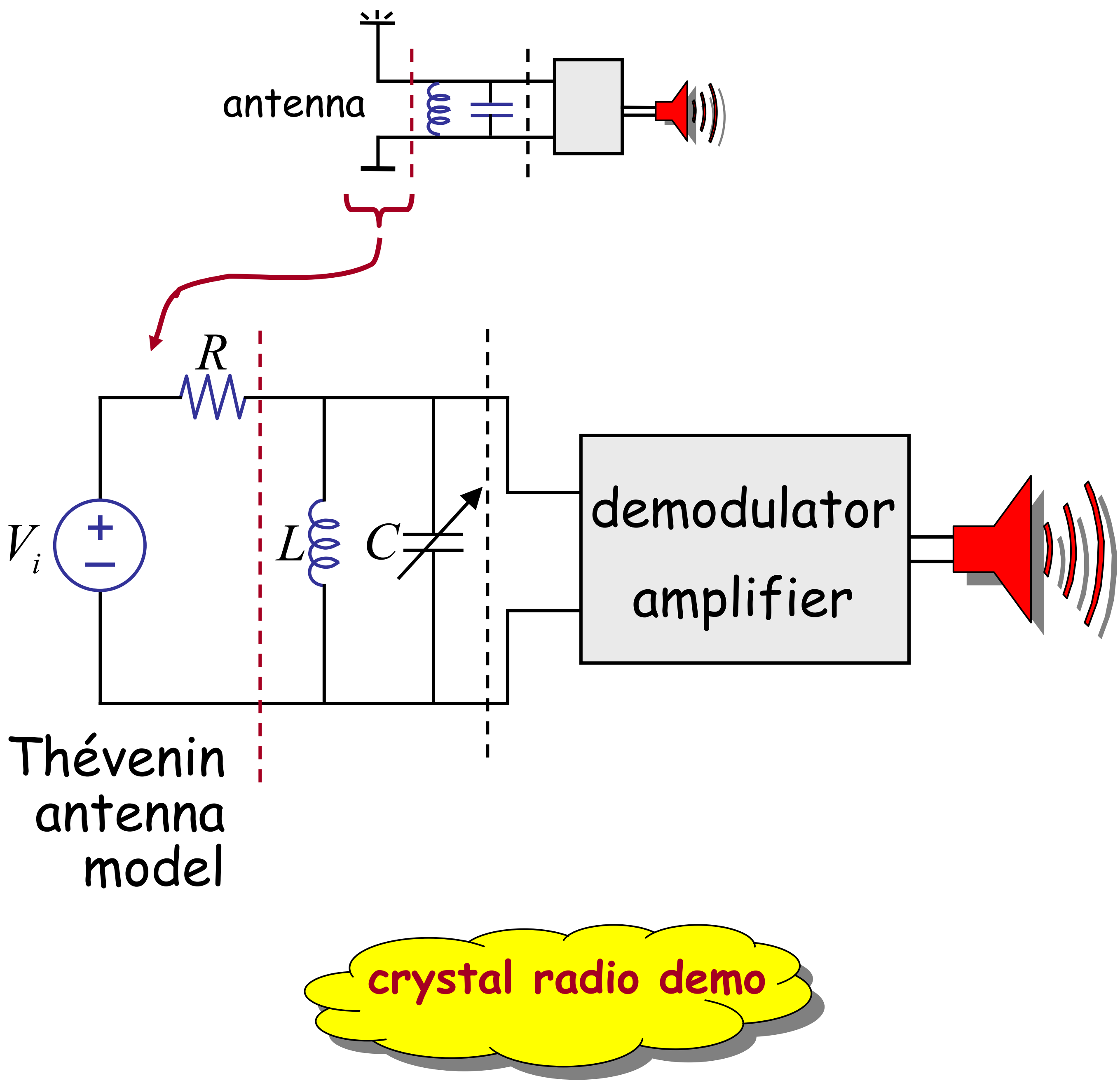
Another example:



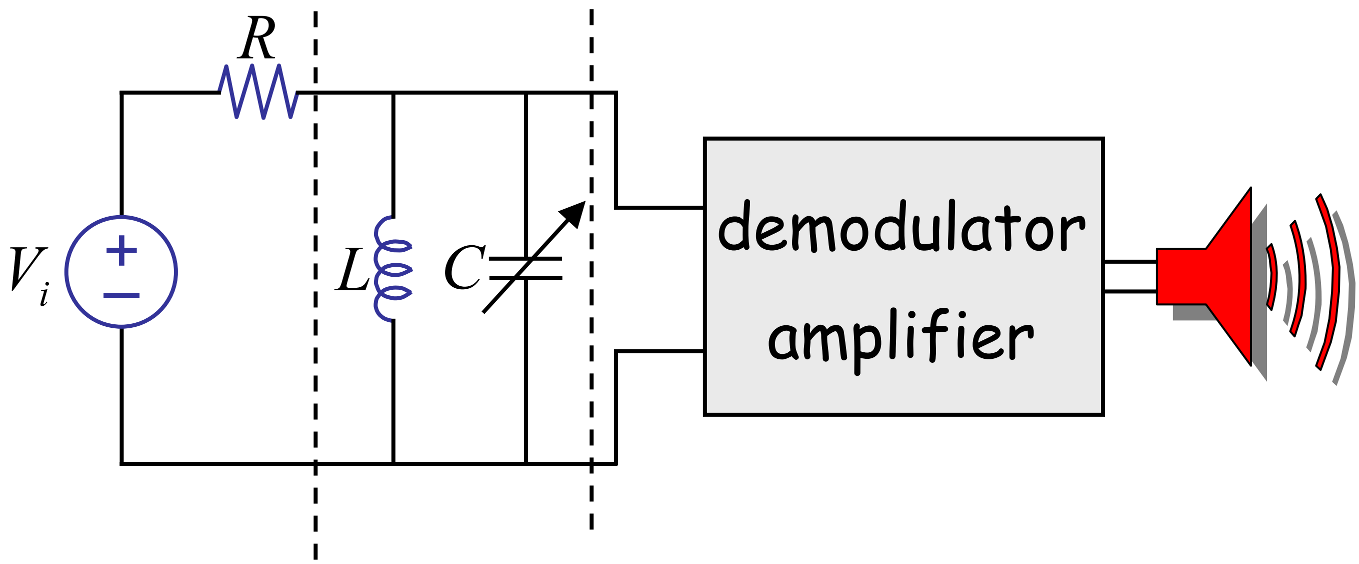


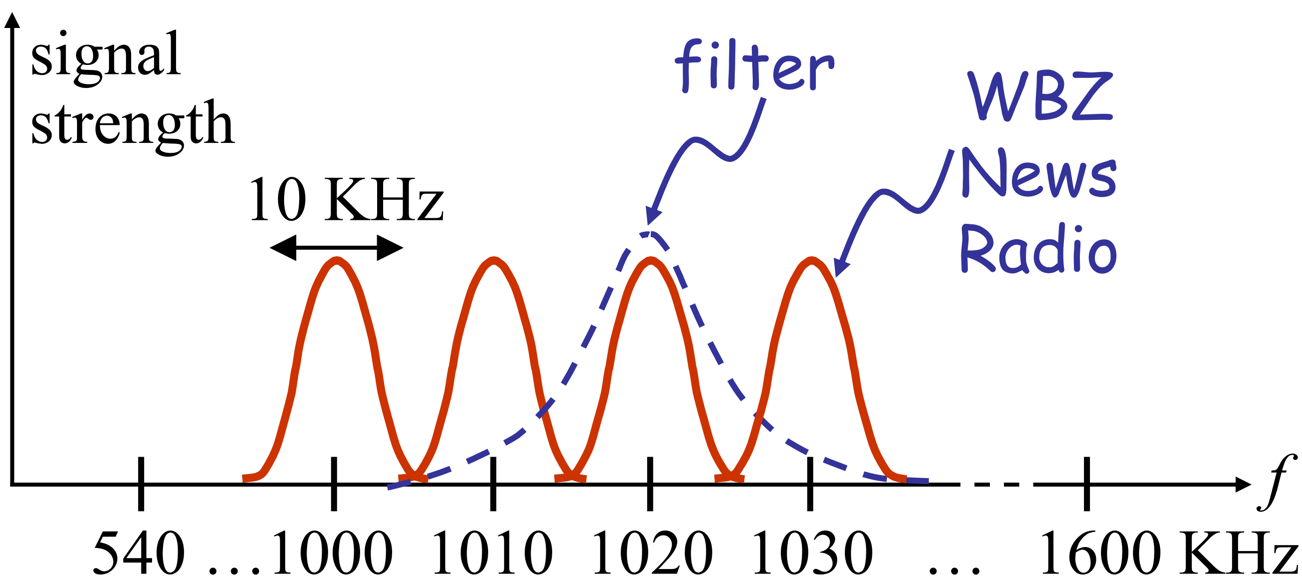
Application: see AM radio coming up shortly

AM Radio Receiver



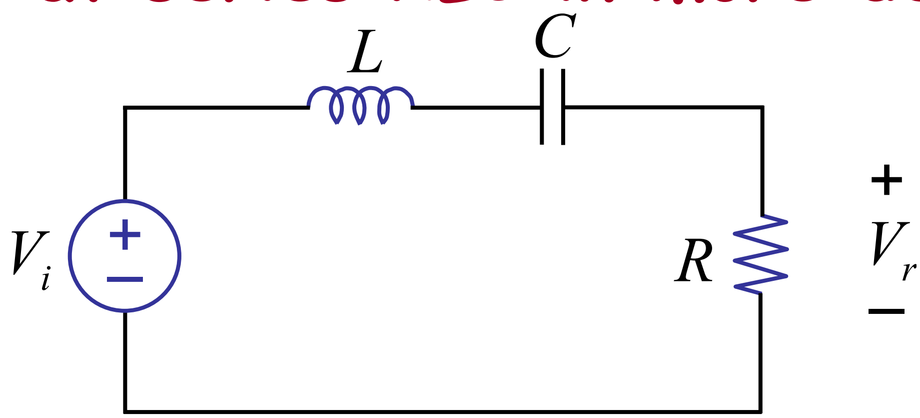
AM Receiver



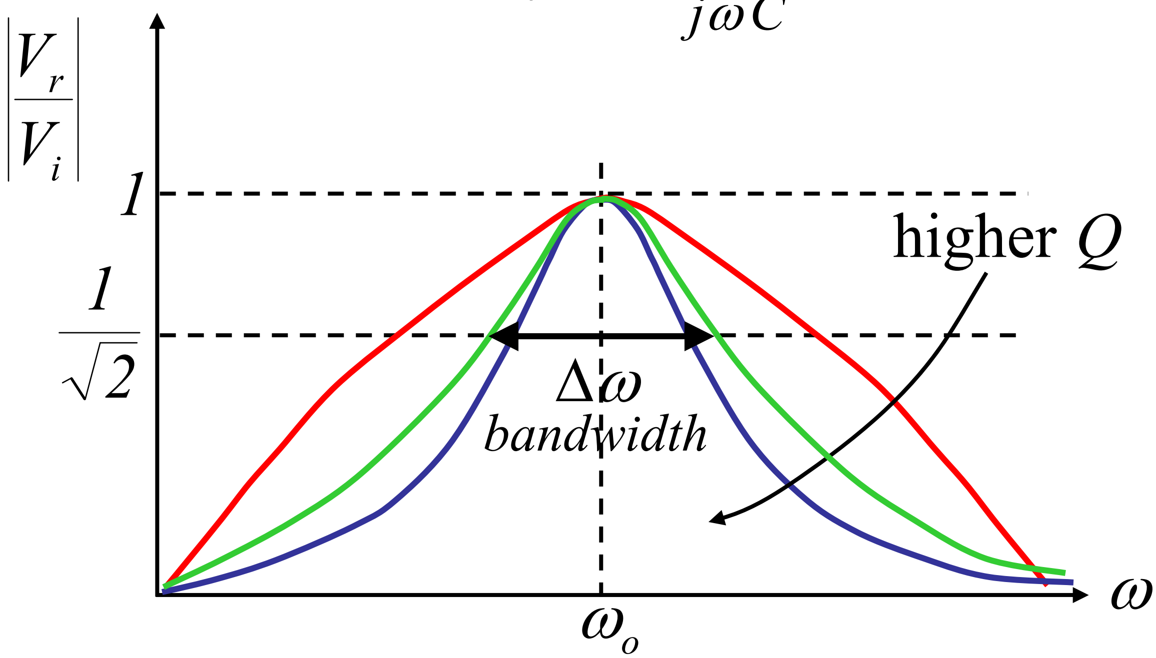


"Selectivity" important —
relates to a parameter "Q" for the filter. Next…

Selectivity:
Look at series RLC in more detail



Recall, $$\frac{V_r}{V_i} = \frac{R}{R + j\omega L + \dfrac{1}{j\omega C}}$$



Define $Q = \dfrac{\omega_o}{\Delta\omega}$ *quality factor*

high Q $\Rightarrow$ more selective

Quality Factor Q

$$Q = \frac{\omega_o}{\Delta\omega}$$

ω_o:

$$\frac{V_r}{V_i} = \frac{R}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1}{1 + j\underbrace{\left(\omega\dfrac{L}{R} - \dfrac{1}{\omega CR}\right)}_{\text{at } \omega_o\ = 0}}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$\Delta\omega$?

Quality Factor Q

$$Q = \frac{\omega_o}{\Delta\omega}$$

$\Delta\omega$:

Note that abs magnitude is $\frac{1}{\sqrt{2}}$

when $$\frac{V_r}{V_i} = \frac{1}{1 + j\left(\omega\frac{L}{R} - \frac{1}{\omega CR}\right)} = \frac{1}{1 \pm j1}$$

i.e. when $$\frac{\omega L}{R} - \frac{1}{\omega CR} = \pm 1$$

$$\omega^2 \mp \frac{\omega R}{L} - \frac{1}{LC} = 0$$

Looking at the roots of both equations,

$$\omega_1 = \frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}} \qquad \omega_2 = -\frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

$$\Delta\omega = \omega_1 - \omega_2 = \frac{R}{L}$$

Quality Factor Q

$$Q = \frac{\omega_o}{\Delta\omega}$$

$$Q = \frac{\omega_o}{\frac{R}{L}} = \frac{\omega_o L}{R} \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

The lower the R (for series R),
the sharper the peak

Quality Factor Q

Another way of looking at Q :

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per cycle}}$$

$$= 2\pi \frac{\frac{1}{2} L \left| I_r \right|^2}{\frac{1}{2} \left| I_r \right|^2 R \frac{2\pi}{\omega_0}}$$

$$Q = \frac{\omega_o L}{R}$$