Incremental Analysis

Review

Nonlinear Analysis

- Analytical method
- Graphical method

Today

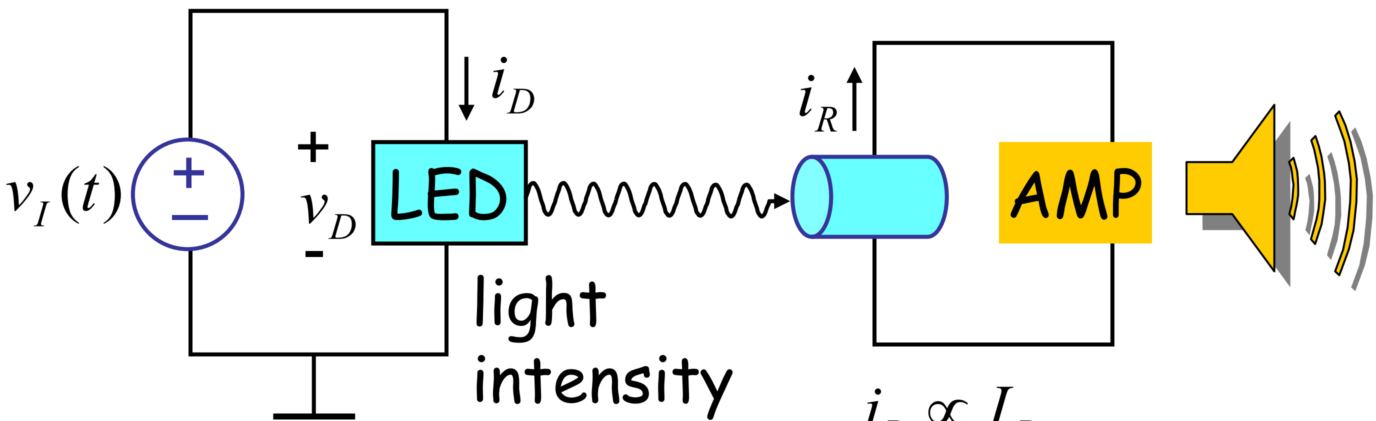
- Incremental analysis

Reading: Section 4.5

Method 3: Incremental Analysis

Motivation: music over a light beam

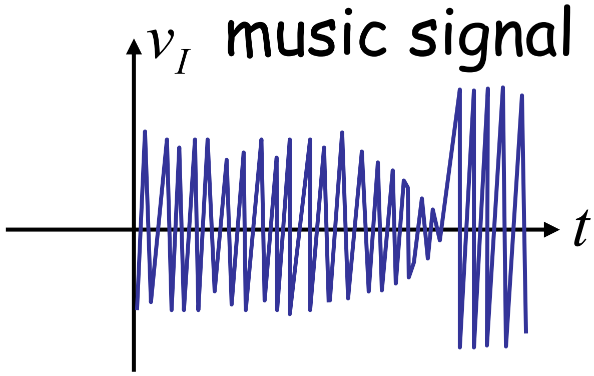
Can we pull this off?



light intensity $I_D \propto i_D$

$i_R \propto I_R$

light intensity I_R in photoreceiver



LED: Light Emitting expoDweep ☺

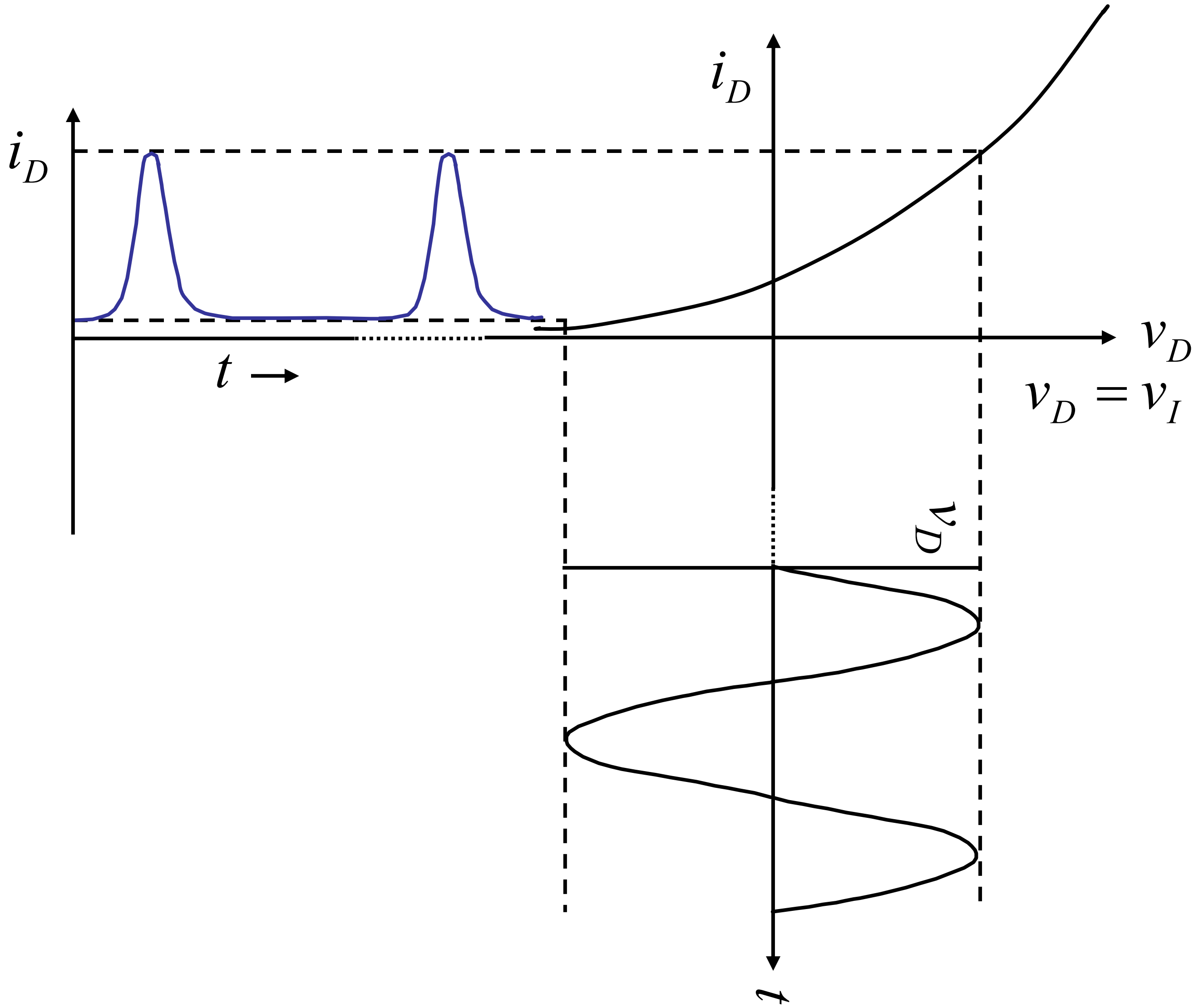
$v_I(t) \longrightarrow i_D(t)$ ⇝ light ⇝ $i_R(t) \longrightarrow$ sound

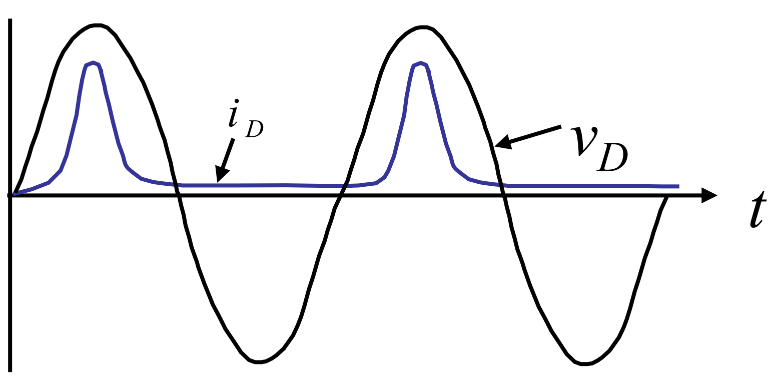
nonlinear

linear

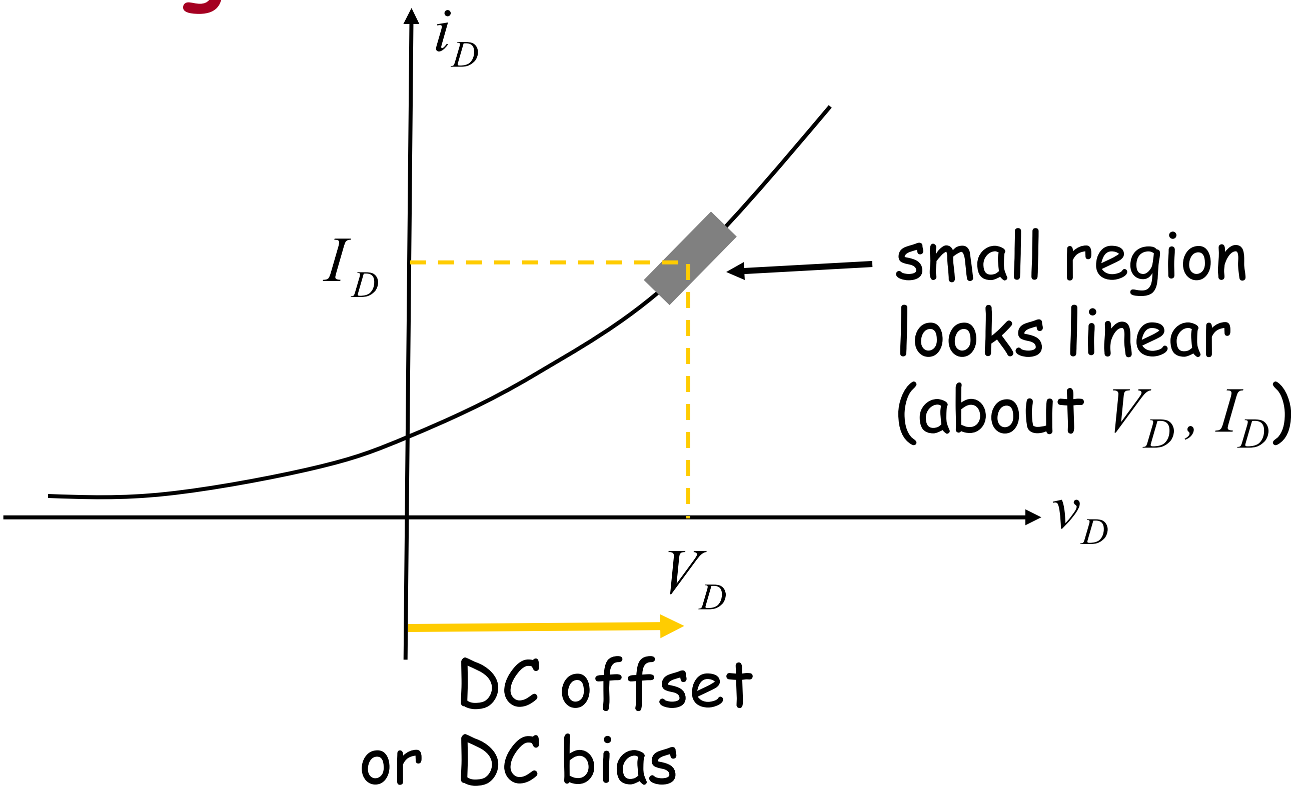
problem! will result in distortion

Problem:
The LED is nonlinear ➡ distortion

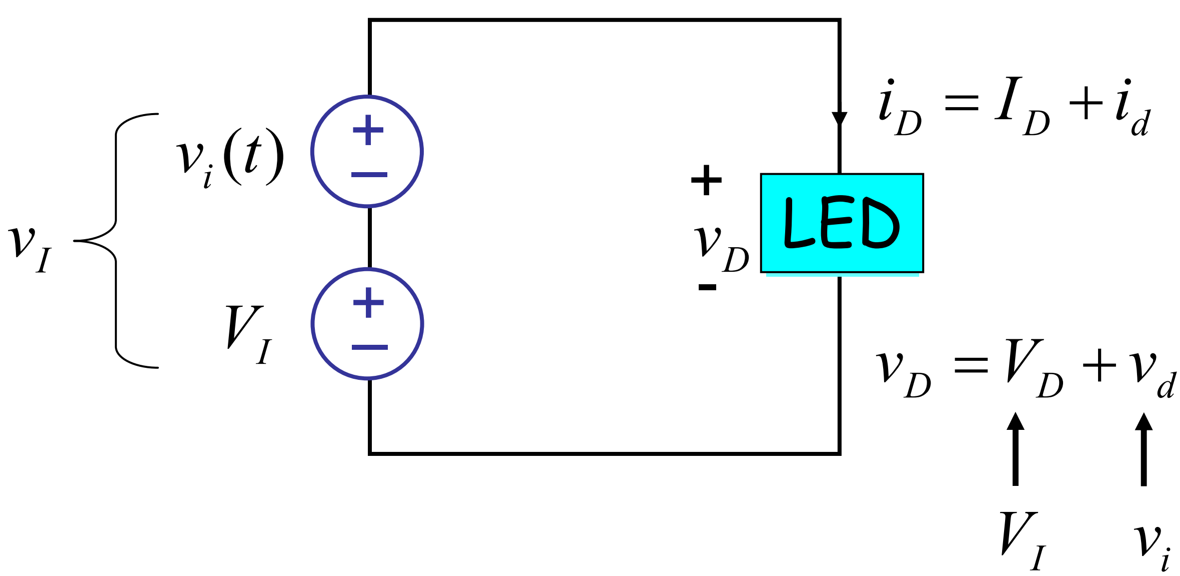




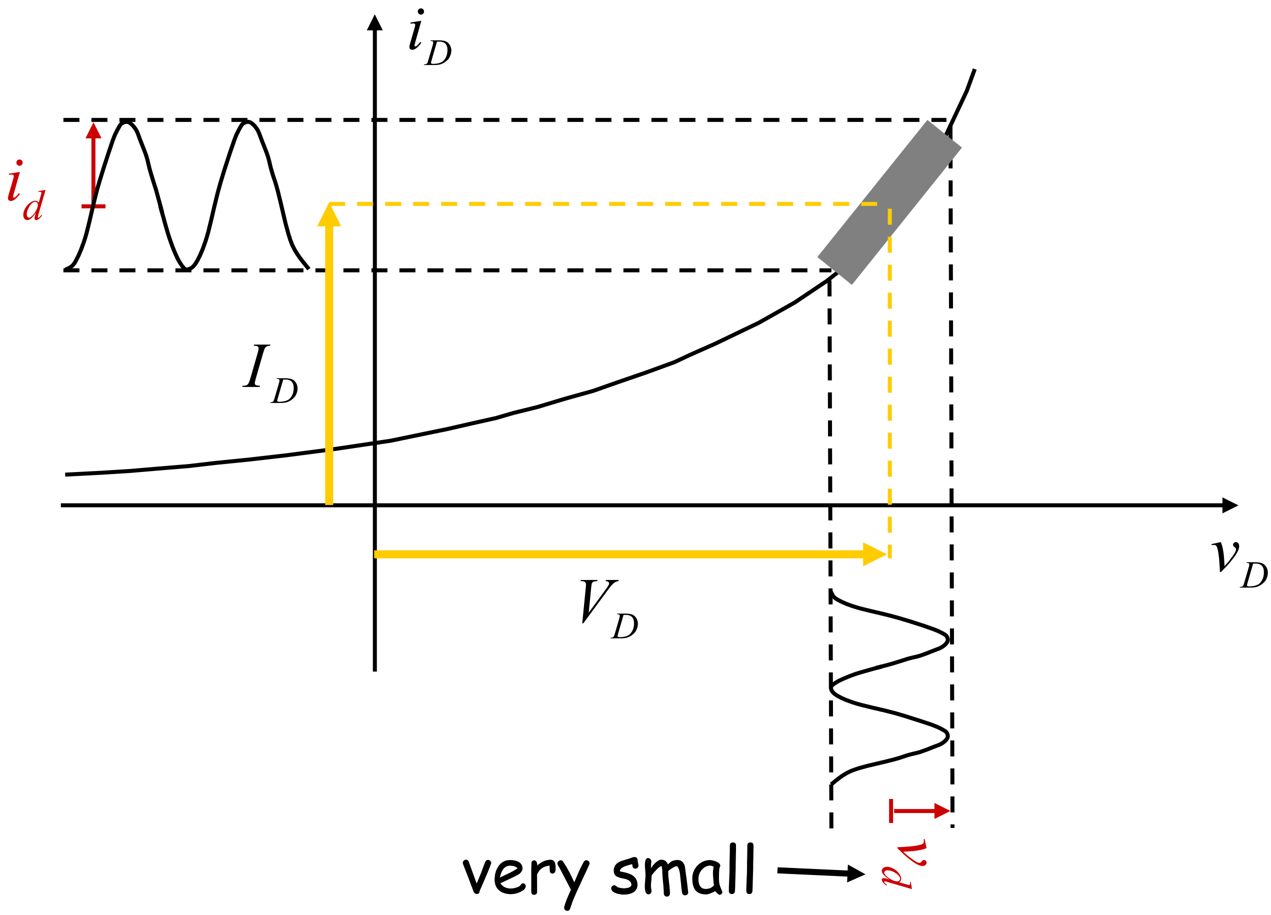
Insight:



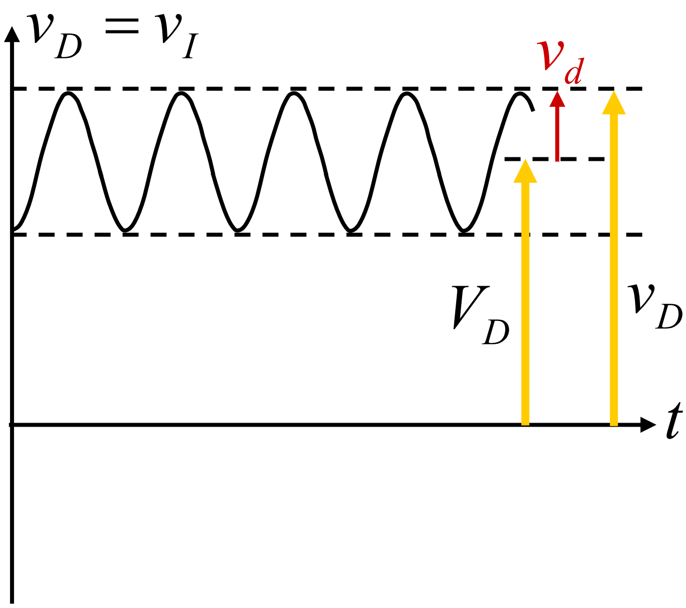
Trick:

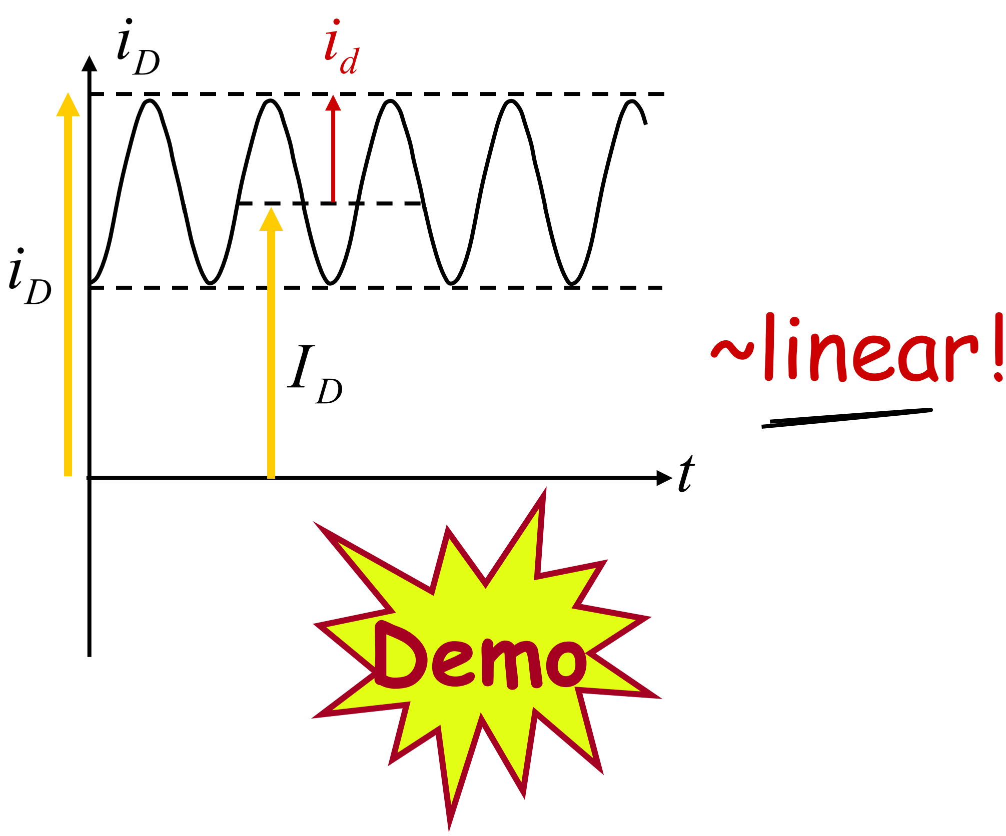


Result



Result





The incremental method:
(or small signal method)

1. Operate at some DC offset or bias point V_D, I_D.
2. Superimpose small signal v_d (music) on top of V_D.
3. Response i_d to small signal v_d is approximately linear.

Notation:

$$i_D = I_D + i_d$$

total variable — DC offset — small superimposed signal

What does this mean mathematically?

Or, why is the small signal response linear?

nonlinear

$$i_D = f(v_D)$$

large DC

We replaced

v_d

$$v_D = V_D + \Delta v_D$$

increment about V_D

using Taylor's Expansion to expand $f(v_D)$ near $v_D = V_D$:

$$i_D = f(V_D) + \left. \frac{df(v_D)}{dv_D} \right|_{v_D = V_D} \cdot \Delta v_D$$

$$\cancel{+ \frac{1}{2!} \left. \frac{d^2 f(v_D)}{dv_D^2} \right|_{v_D = V_D}} \cdot \Delta v_D^2 + \cdots$$

neglect higher order terms because Δv_D is small

$$i_D \approx \underbrace{f(V_D)}_{\substack{\text{constant} \\ \text{w.r.t. } \Delta v_D}} + \underbrace{\left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D=V_D}}_{\substack{\text{constant w.r.t. } \Delta v_D \\ \text{slope at } V_D,\, I_D}} \cdot \Delta v_D$$

We can write

$$X: \; I_D + \Delta i_D \approx f(V_D) + \left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D=V_D} \cdot \Delta v_D$$

equating DC and time-varying parts,

$$I_D = f(V_D) \longrightarrow \text{operating point}$$

$$\Delta i_D = \underbrace{\left.\frac{d\,f(v_D)}{dv_D}\right|_{v_D=V_D}}_{\text{constant w.r.t. } \Delta v_D} \cdot \Delta v_D$$

so, $\Delta i_D \propto \Delta v_D$

By notation,

$$\Delta i_D = i_d$$
$$\Delta v_D = v_d$$

In our example,

$$i_D = a\,e^{b v_D}$$

From ⓧ: $I_D + i_d \approx a\,e^{bV_D} + a\,e^{bV_D} \cdot b \cdot v_d$

Equate DC and incremental terms,

$\boxed{I_D = a\,e^{bV_D}}$ → operating point
aka bias pt.
aka DC offset

$$i_d = a\,e^{bV_D}\,b \cdot v_d$$

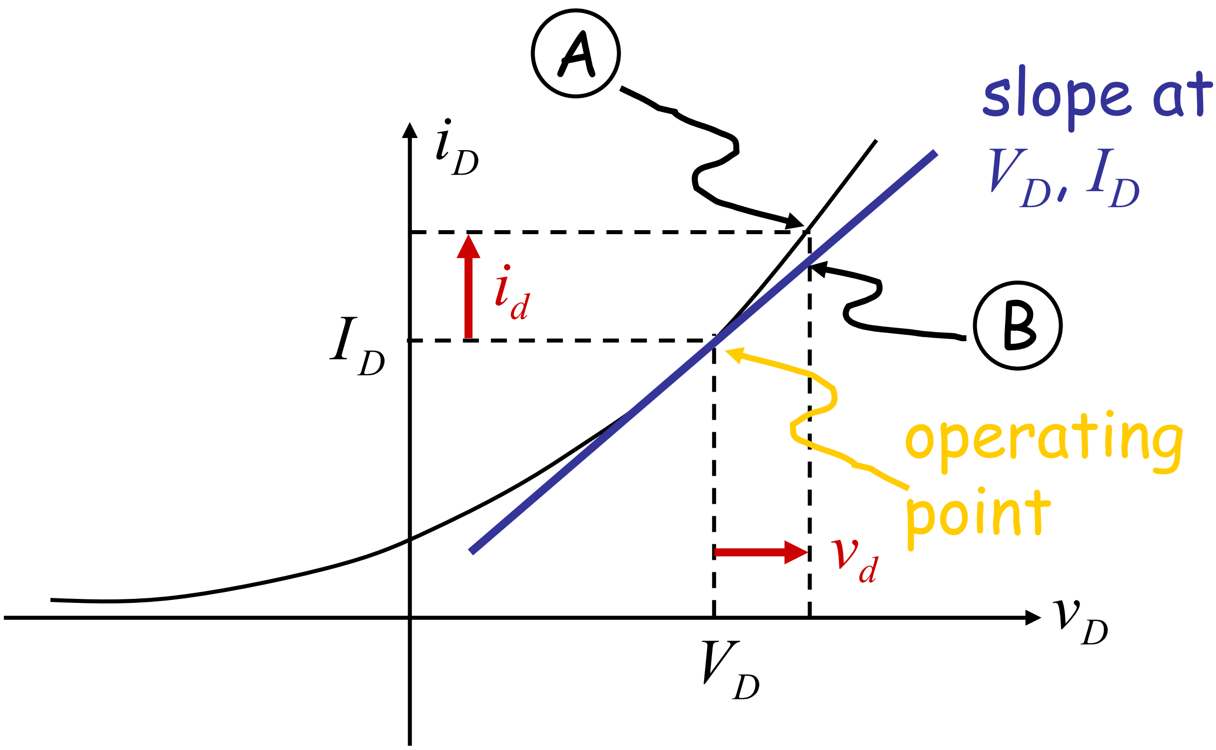
$i_d = \underbrace{I_D \cdot b}_{\text{constant}} \cdot v_d$ → small signal behavior

constant → linear!

Graphical interpretation

$$I_D = a\, e^{bV_D} \longrightarrow \text{operating point}$$

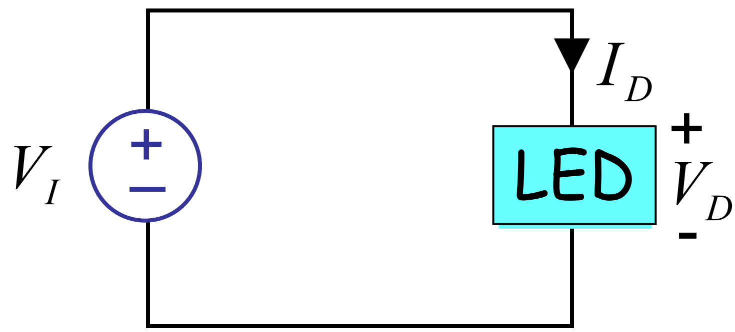
$$i_d = I_D \cdot b \cdot v_d$$



we are
approximating
A with B

We saw the small signal → graphically, mathematically, now, circuit

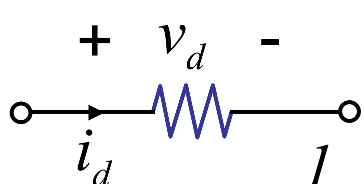
Large signal circuit:



$$\boxed{I_D = a\, e^{bV_D}}$$

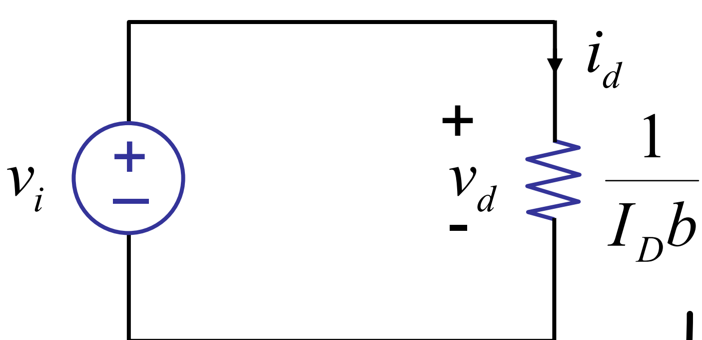
Small signal response: $i_d = I_D\, b\, v_d$

behaves like:



$$R = \frac{1}{I_D\, b}$$

small signal circuit:



Linear!