6.002 CIRCUITS AND ELECTRONICS

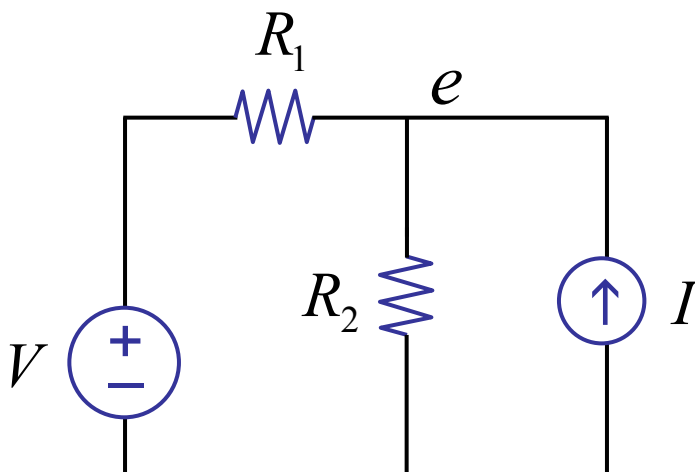Superposition, Thévenin and Norton

Review

Circuit Analysis Methods

- KVL: KCL: VI

$$\sum_{loop} V_i = 0 \qquad \sum_{node} I_i = 0$$

- Circuit composition rules

- Node method – the workhorse of 6.002
  KCL at nodes using V's referenced
  from ground
  (KVL implicit in "$\left(e_i - e_j \right) G$")
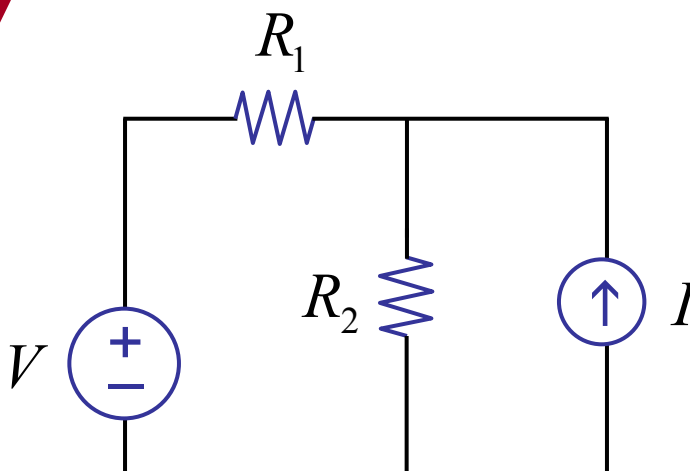
Linearity

Consider

Write node equations –

$$\frac{e-V}{R_1} + \frac{e}{R_2} - I = 0$$

Notice:
linear in e, V, I

No eV, VI
terms

Linearity

Consider

Write node equations --

$$\frac{e-V}{R_1} + \frac{e}{R_2} - I = 0 \qquad \text{linear in } e, V, I$$

Rearrange --

$$\left[\frac{1}{R_1} + \frac{1}{R_2}\right]e = \frac{V}{R_1} + I$$

conductance   node   linear sum
matrix   voltages of sources

$$G \qquad\qquad e = S$$

Linearity

Write node equations --

$$\frac{e-V}{R_1} + \frac{e}{R_2} - I = 0 \qquad \text{linear in } e, V, I$$

Rearrange --

$$\left[\frac{1}{R_1} + \frac{1}{R_2}\right]e = \frac{V}{R_1} + I$$

conductance   node   linear sum
matrix   voltages   of sources

$$G \qquad\qquad e \;=\; S$$

or $\quad e = \dfrac{R_2}{R_1 + R_2} V + \dfrac{R_1 R_2}{R_1 + R_2} I$

$$e = a_1 V_1 + a_2 V_2 + \ldots + b_1 I_1 + b_2 I_2 + \ldots$$

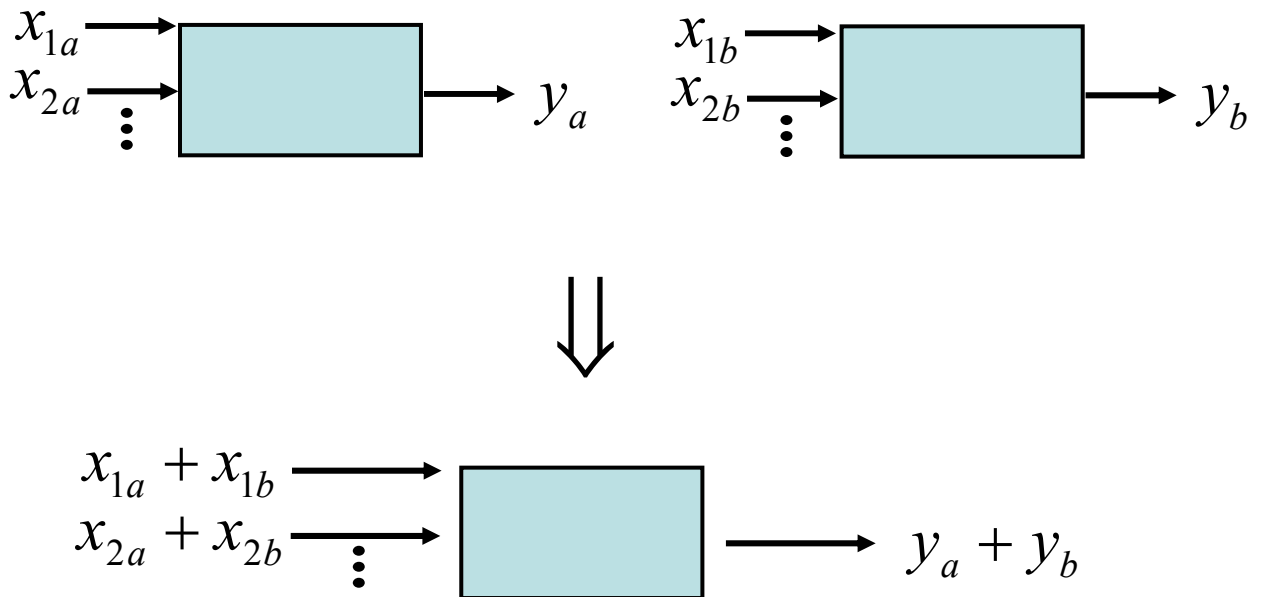Linear!

Linearity $\Longrightarrow$ Homogeneity
Superposition

Linearity $\implies$ Homogeneity
Superposition

Homogeneity

$$x_1 \longrightarrow$$
$$x_2 \longrightarrow$$
$$\vdots$$
$$\boxed{} \longrightarrow y$$

$$\Downarrow$$

$$\alpha x_1 \longrightarrow$$
$$\alpha x_2 \longrightarrow$$
$$\vdots$$
$$\boxed{} \longrightarrow \alpha y$$
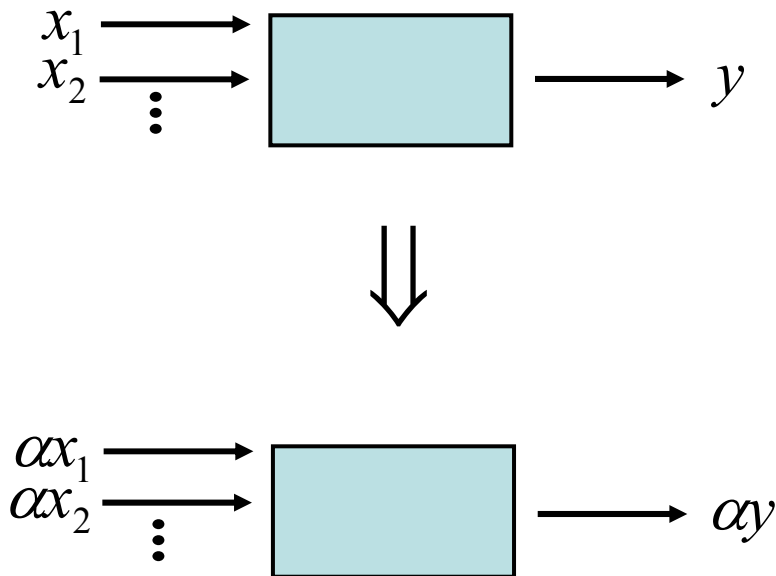
Linearity $\Longrightarrow$ Homogeneity
Superposition

Superposition

x_{1a} →
x_{2a} → ⋮ → y_a

x_{1b} →
x_{2b} → ⋮ → y_b

$\Downarrow$

$x_{1a} + x_{1b}$ →
$x_{2a} + x_{2b}$ → ⋮ → $y_a + y_b$

Linearity $\Longrightarrow$ Homogeneity
Superposition

Specific superposition example:

Method 4: Superposition method

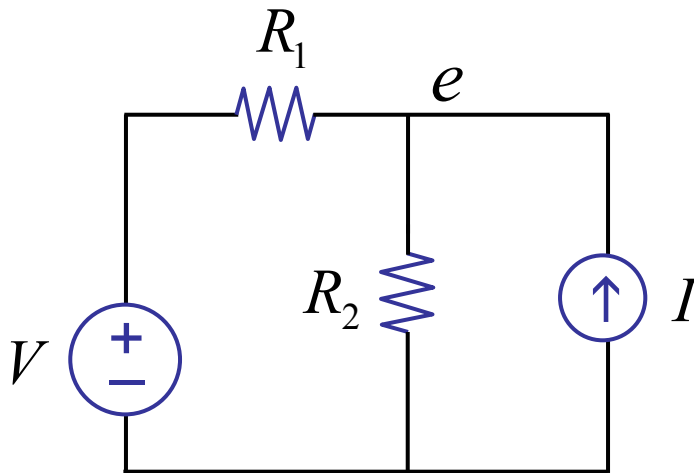The output of a circuit is determined by summing the responses to each source acting alone.
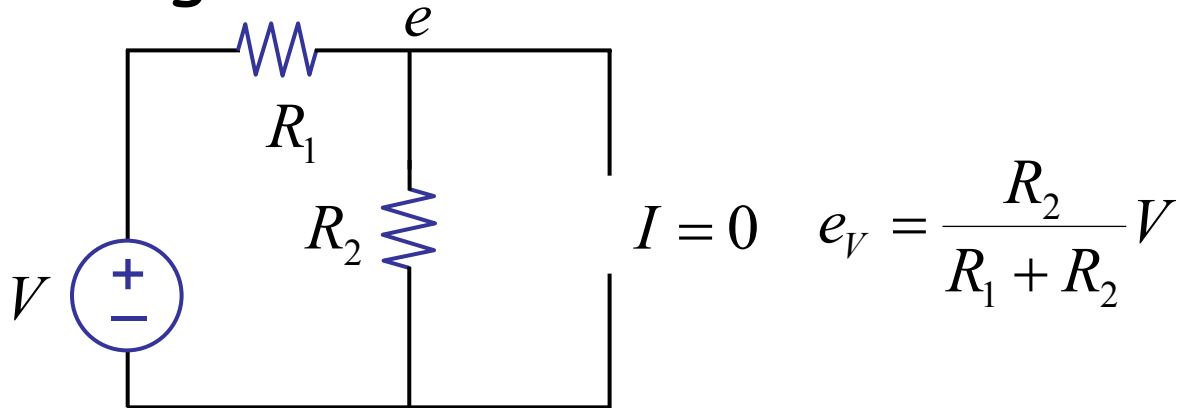
independent sources only

$V = 0$ i $+$ v $-$

⟶

i $+$ v $-$
short

$I = 0$ i $+$ v $-$

⟶

i $+$ v $-$
open
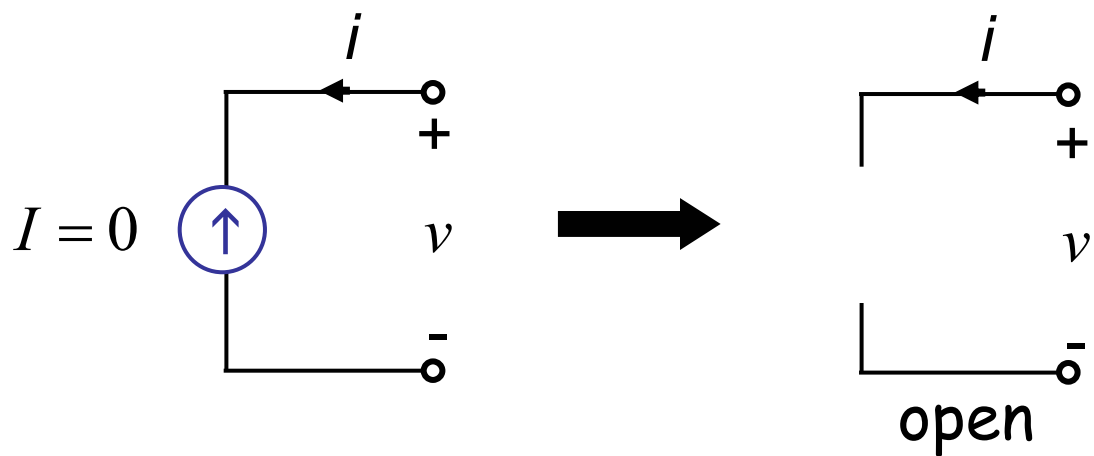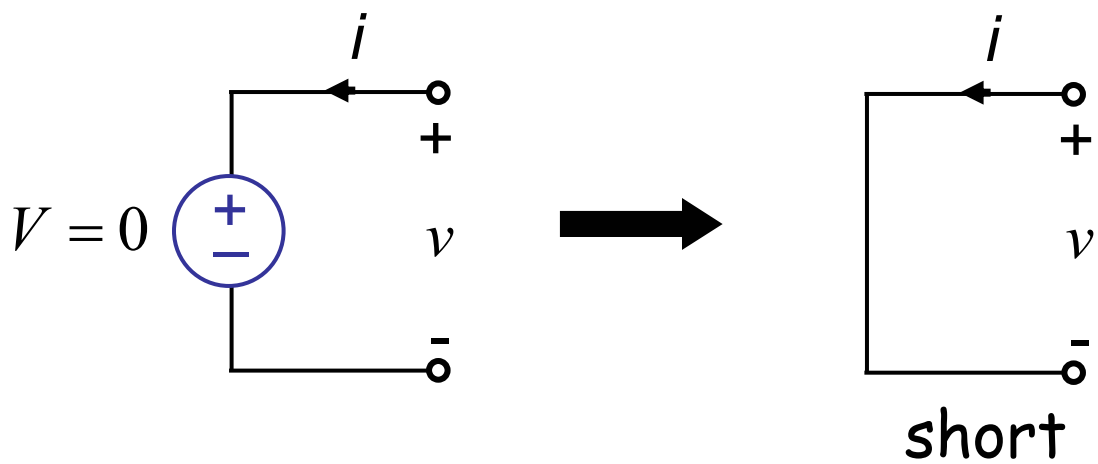
Back to the example
Use superposition method

Back to the example
Use superposition method

V acting alone

$$I = 0 \quad e_V = \frac{R_2}{R_1 + R_2} V$$

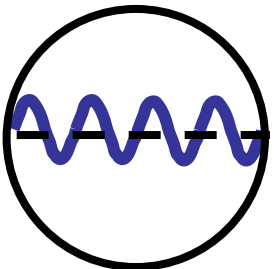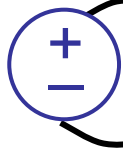### I acting alone
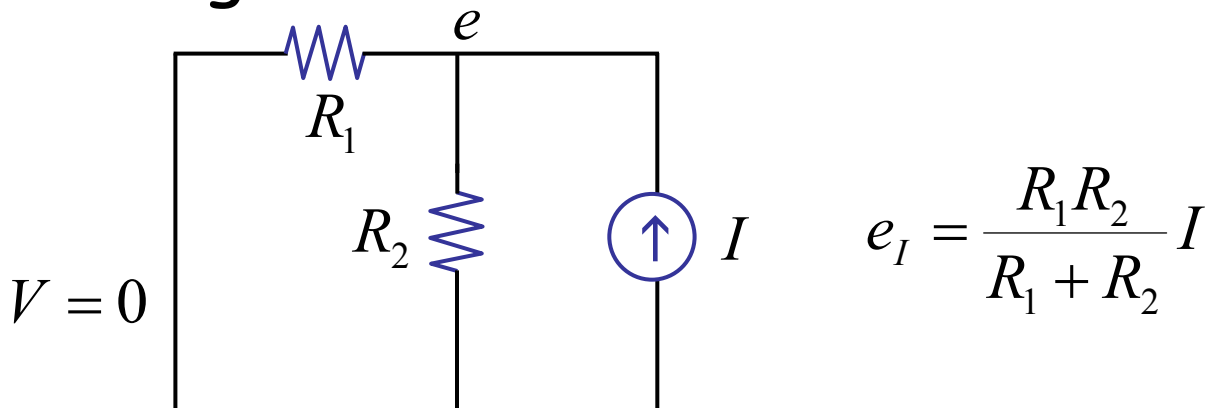
$$e_I = \frac{R_1 R_2}{R_1 + R_2} I$$

sum $\longrightarrow$ superposition

$$e = e_V + e_I = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I$$

Voilà !

Demo

constant

salt water
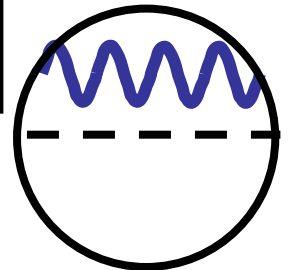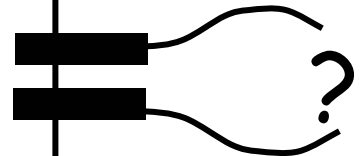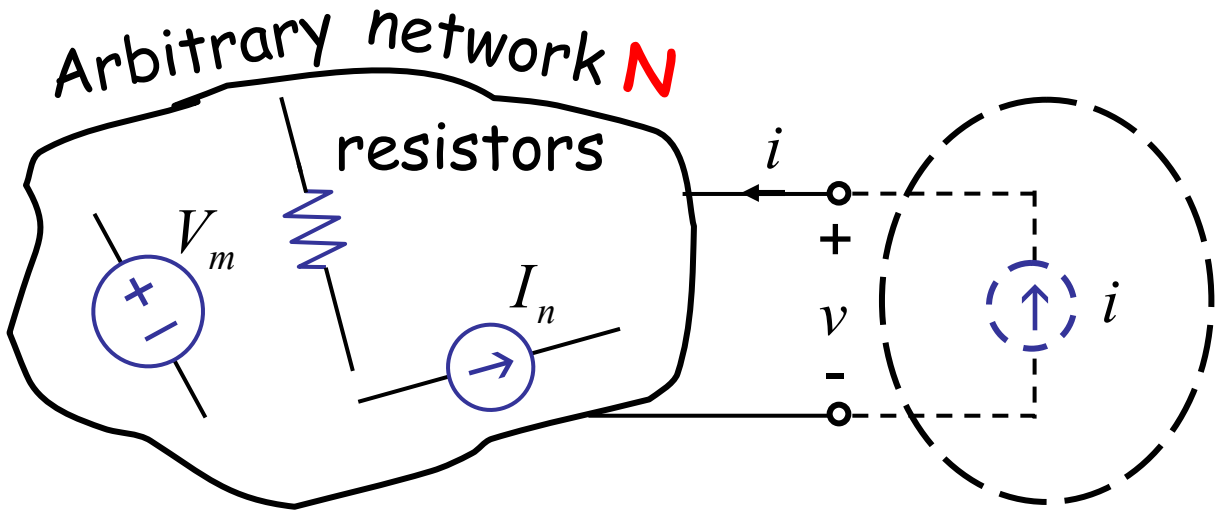
sinusoid

?
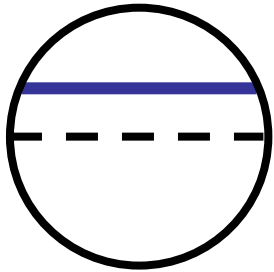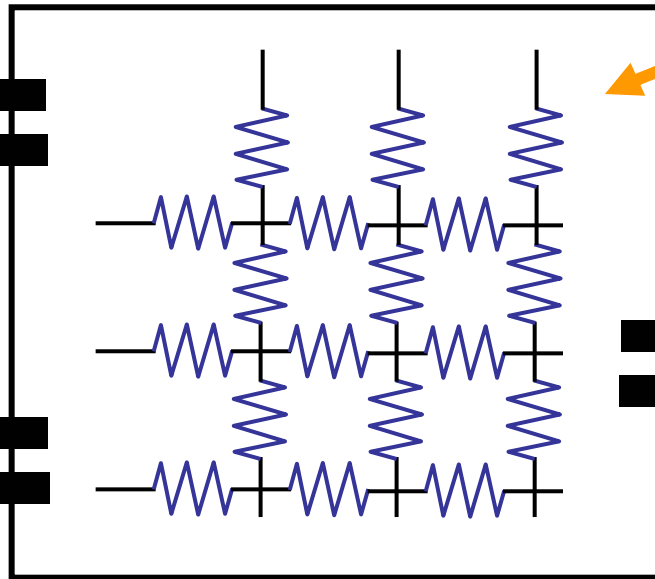
output shows superposition

Yet another method…

Consider

Arbitrary network **N**

resistors

V_m

I_n

i

+

v

-

i

By superposition

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$

no units

resistance units

also independent of external excitement & behaves like a resistor

$\underbrace{\text{By setting}}$

By setting
$\forall_n I_n = 0,$ $\forall_m V_m = 0,$
  $i = 0$ $i = 0$

All
$\forall_n I_n = 0,$
$\forall_m V_m = 0$

independent of external excitation and behaves like a voltage " v_{TH} "

Or

$$v = v_{TH} + R_{TH}i$$

As far as the external world is concerned (for the purpose of I-V relation), "Arbitrary network N" is indistinguishable from:
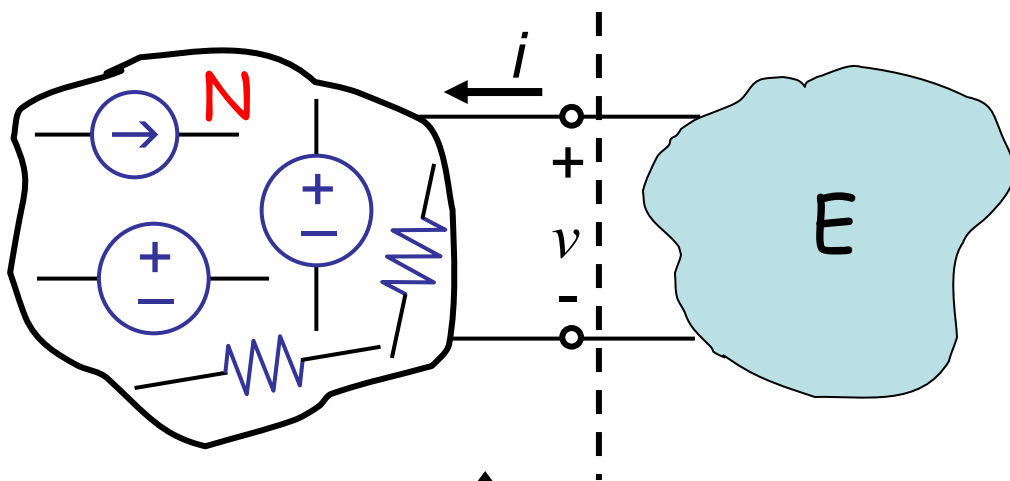

Thévenin equivalent network
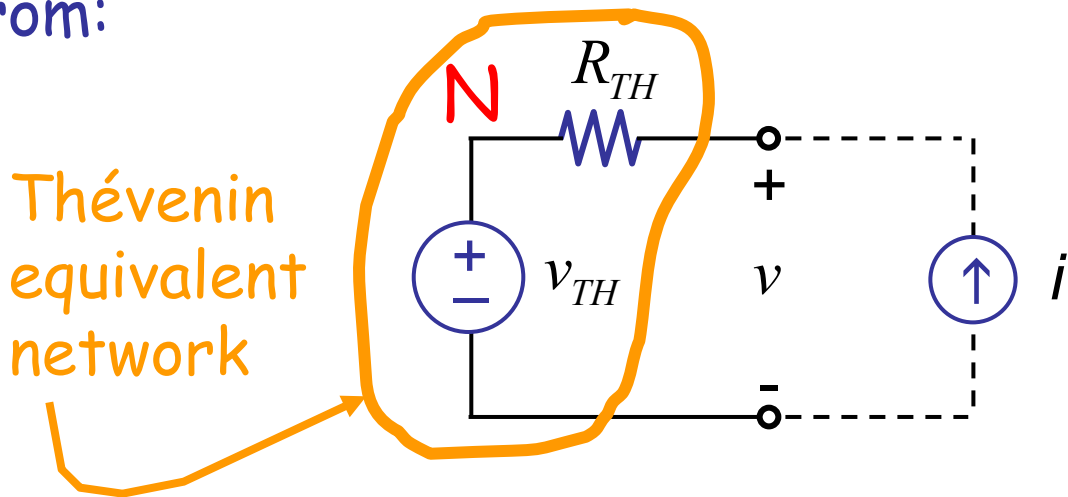
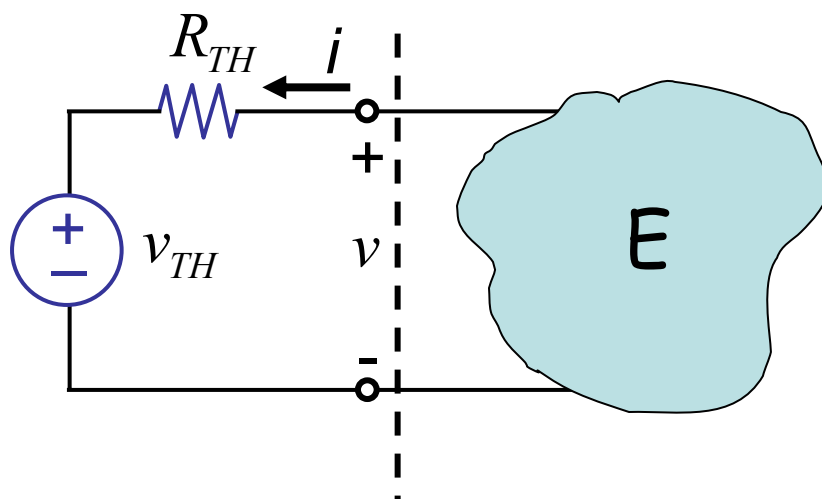v_{TH} ⟶ open circuit voltage at terminal pair (a.k.a. port)

R_{TH} ⟶ resistance of network seen from port (V_m's, I_n's set to 0)
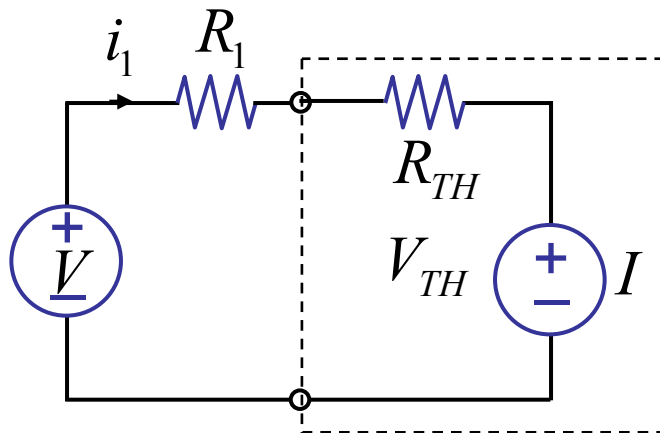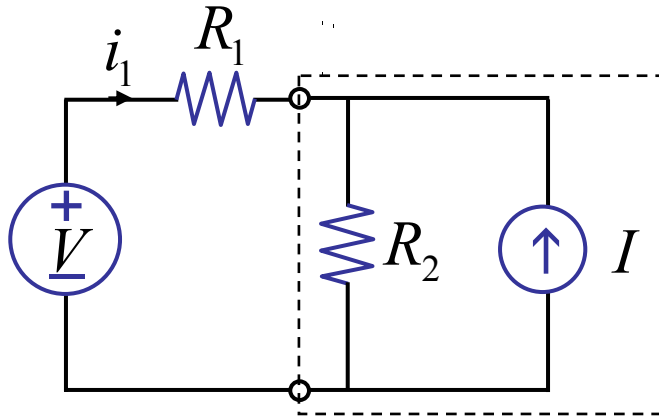
Method 4:
The Thévenin Method

Thévenin equivalent

Replace network N with its Thévenin
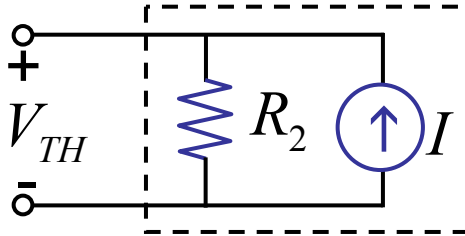equivalent, then solve external network E.

Example:

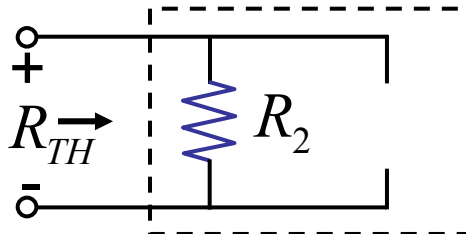$$i_1 = \frac{V - V_{TH}}{R_1 + R_{TH}}$$

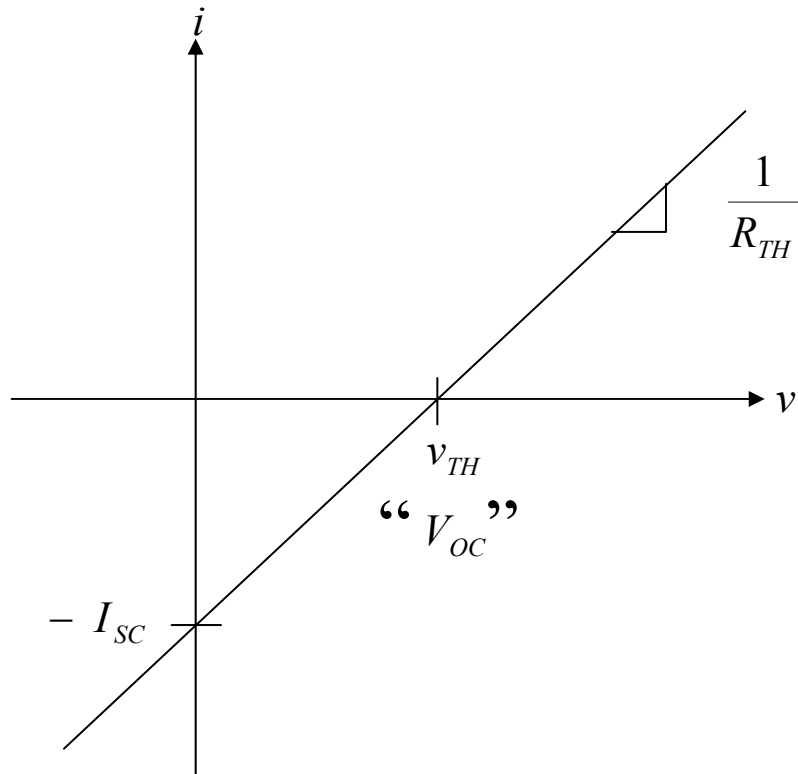# Example:

V_{TH} :

$V_{TH} = IR_2$

R_{TH} :

$R_{TH} = R_2$

Graphically, $\quad v = v_{TH} + R_{TH}i$
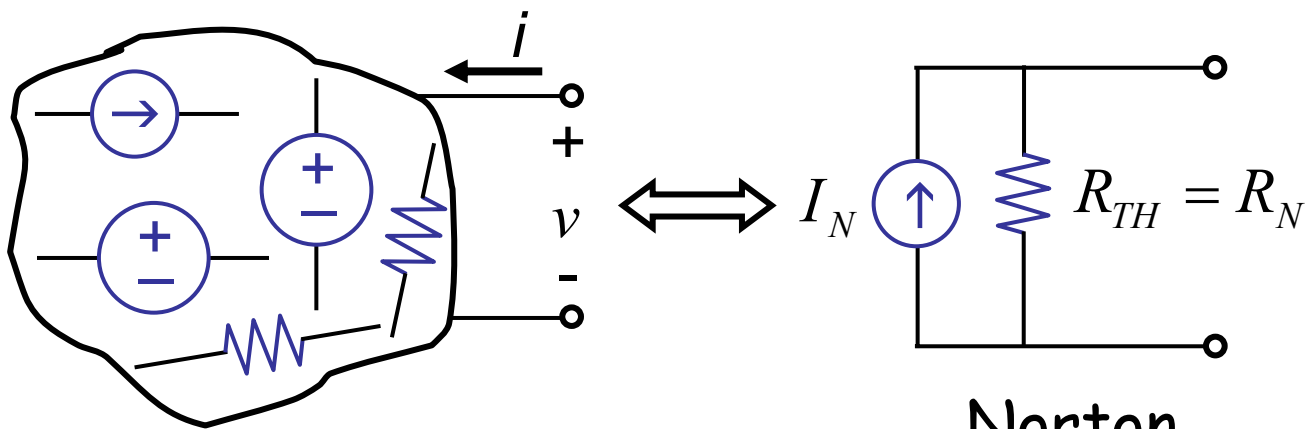

Open circuit $v = v_{TH} \longleftarrow V_{OC}$
$(i \equiv 0)$

Short circuit $i = \dfrac{-v_{TH}}{R_{TH}} \longleftarrow -I_{SC}$
$(v \equiv 0)$

The Norton Method

$$I_N = \frac{V_{TH}}{R_{TH}}$$

Norton equivalent

Summary

■Discretize matter

$$LMD \longrightarrow LCA$$
$$Physics \longrightarrow EE$$

■ R, I, V ┌─────────┐ networks
           │ Linear │
           └─────────┘

■ Analysis methods (linear)
KVL, KCL, I — V
Combination rules
Node method
Superposition
Thévenin
Norton

■ Next
Nonlinear analysis
Discretize voltage

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