

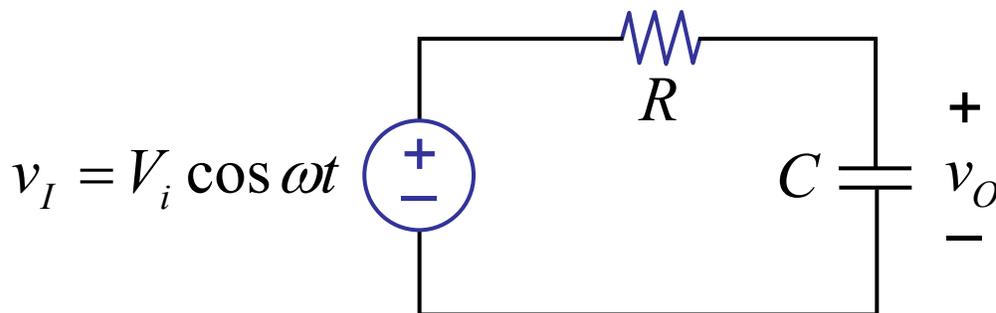
6.002

**CIRCUITS AND
ELECTRONICS**

The Impedance Model

Review

- Sinusoidal Steady State (SSS)
Reading 13.1, 13.2



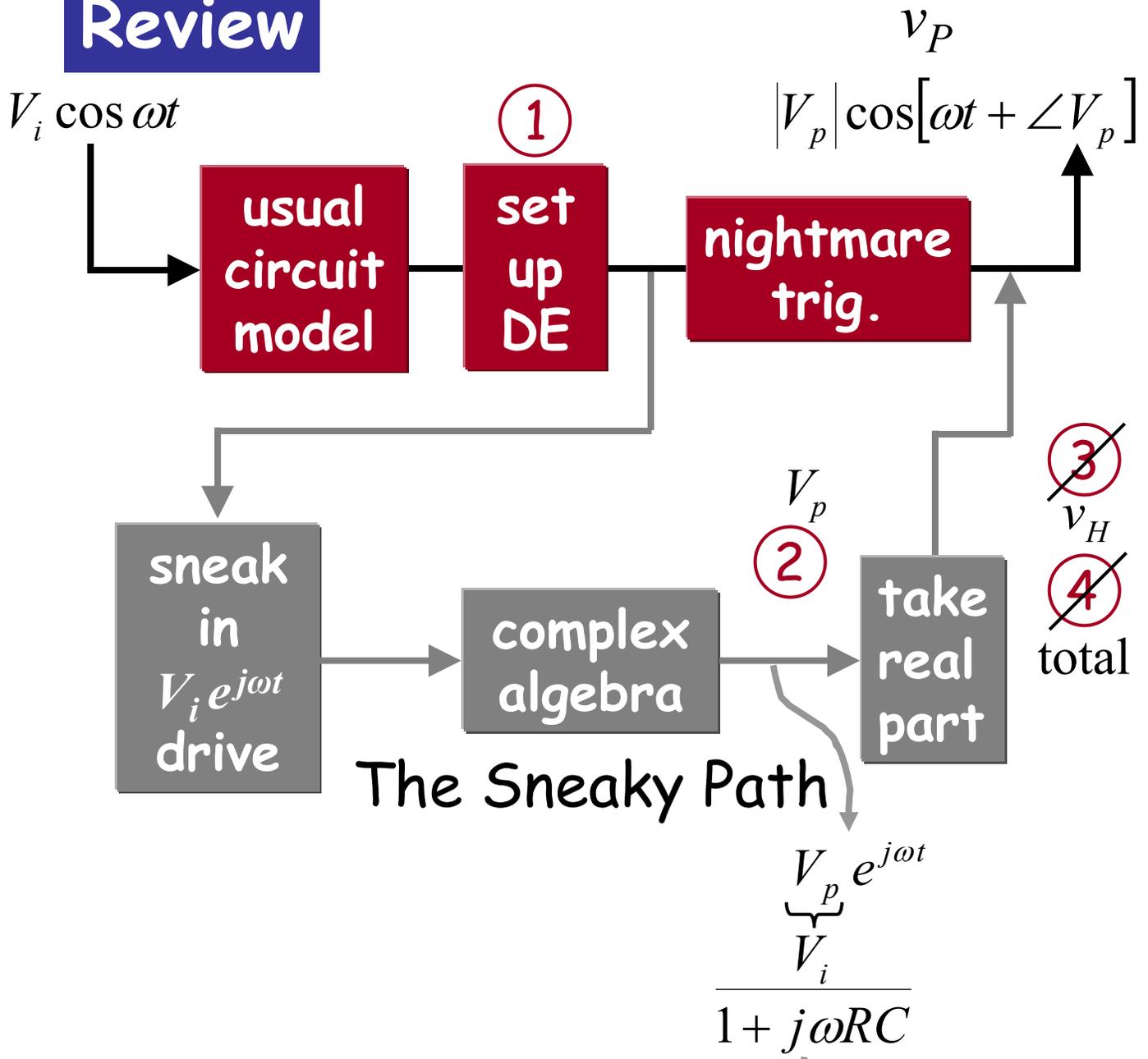
SSS

- Focus on steady state, only care about v_P as v_H dies away.
- Focus on sinusoids.

- Sinusoidal Steady State (SSS)
Reading 13.1, 13.2

Reading: Section 13.3 from course notes.

Review



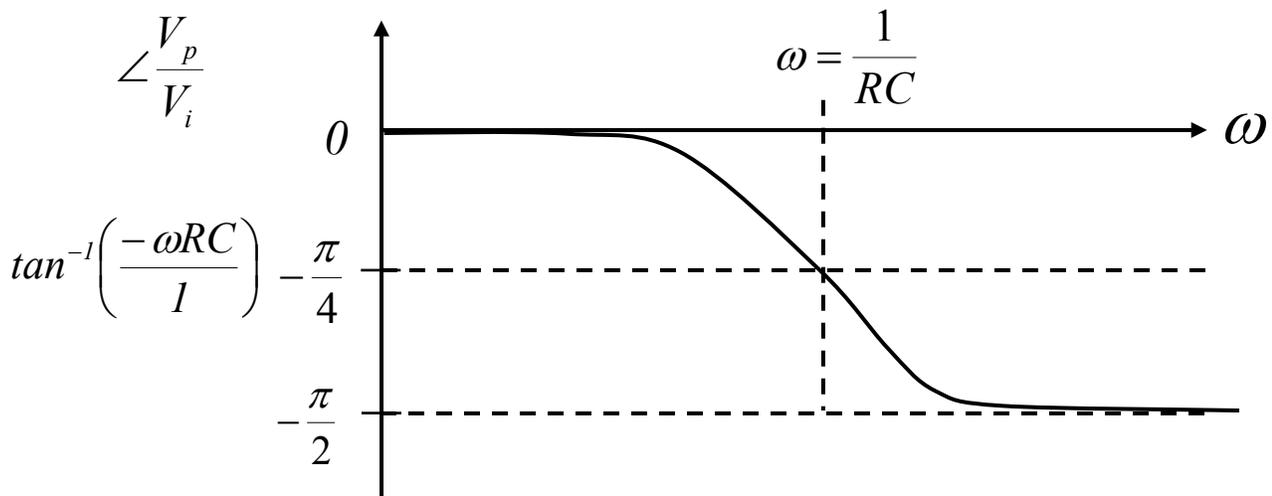
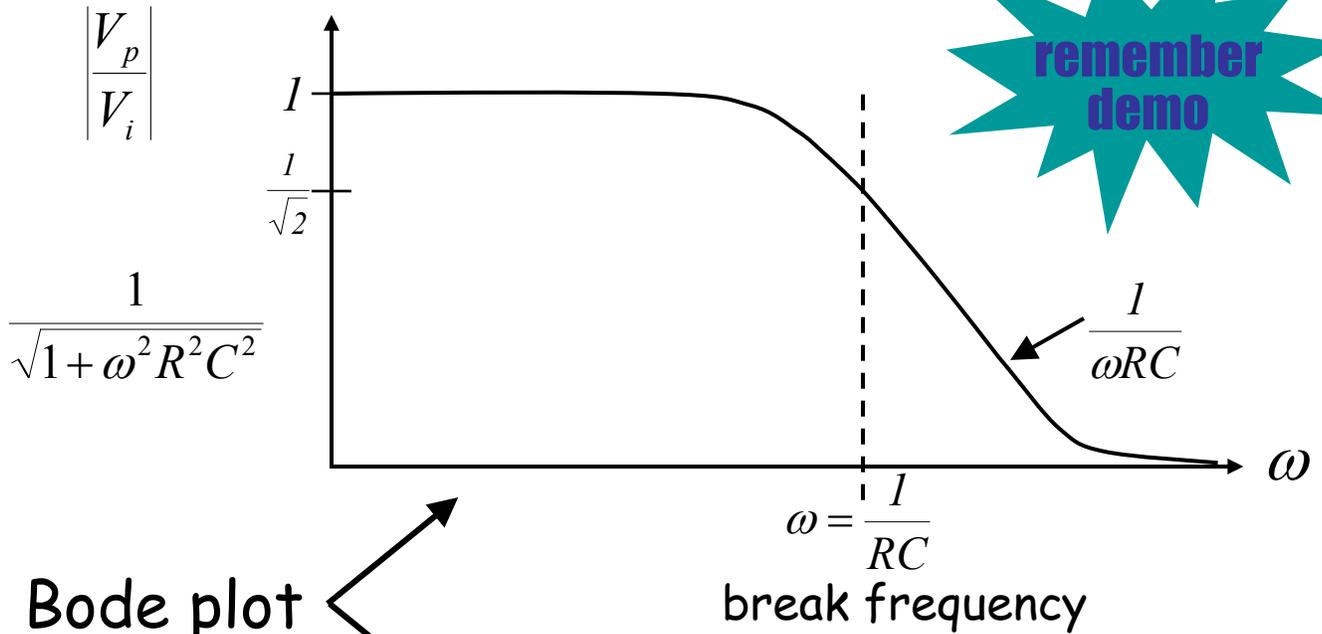
V_p contains all the information we need:

$|V_p|$ Amplitude of output cosine
 $\angle V_p$ phase

Review

$$v_o = |V_p| \cos(\omega t + \angle V_p)$$

$$\frac{V_p}{V_i} = \frac{1}{1 + j\omega RC} = H(j\omega) \quad \text{transfer function}$$



The Frequency View

Is there an even simpler way to get V_p ?

$$V_p = \frac{V_i}{1 + j\omega RC}$$

Divide numerator and denominator by $j\omega C$.

$$V_p = V_i \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}$$

Hmmm... looks like a voltage divider relationship.

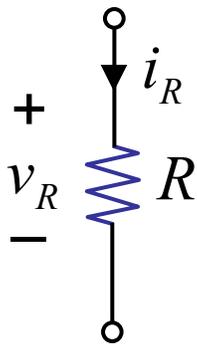
$$V_p = V_i \frac{Z_C}{Z_C + R}$$

Let's explore further...

The Impedance Model

Is there an even simpler way to get V_p ?

Consider:



$$i_R = I_r e^{j\omega t}$$

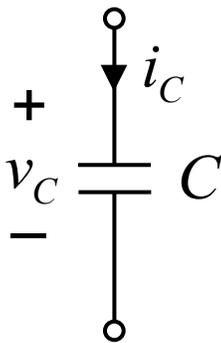
$$v_R = R i_R$$

$$v_R = V_r e^{j\omega t}$$

$$V_r e^{j\omega t} = R I_r e^{j\omega t}$$

Resistor

$$V_r = R I_r$$



$$i_C = I_c e^{j\omega t}$$

$$i_C = C \frac{dv_C}{dt}$$

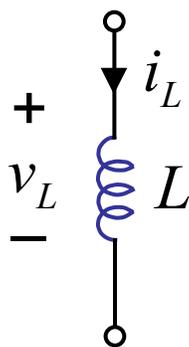
$$v_C = V_c e^{j\omega t}$$

$$I_c e^{j\omega t} = C V_c j\omega e^{j\omega t}$$

Capacitor

$$V_C = \frac{1}{j\omega C} I_C$$

Z_C



$$i_L = I_l e^{j\omega t}$$

$$v_L = L \frac{di_L}{dt}$$

$$v_L = V_l e^{j\omega t}$$

$$V_l e^{j\omega t} = L I_l j\omega e^{j\omega t}$$

Inductor

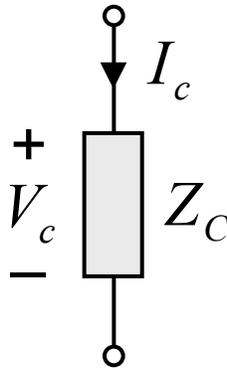
$$V_l = j\omega L I_l$$

Z_L

The Impedance Model

In other words,

capacitor

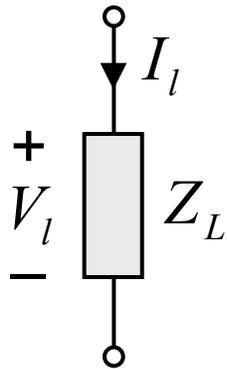


$$V_c = Z_C I_c$$

$$Z_C = \frac{1}{j\omega C}$$

impedance

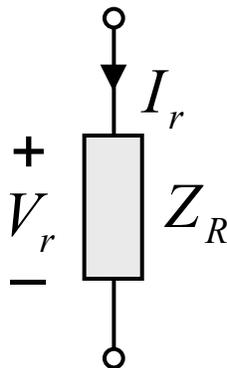
inductor



$$V_l = Z_l I_l$$

$$Z_l = j\omega L$$

resistor

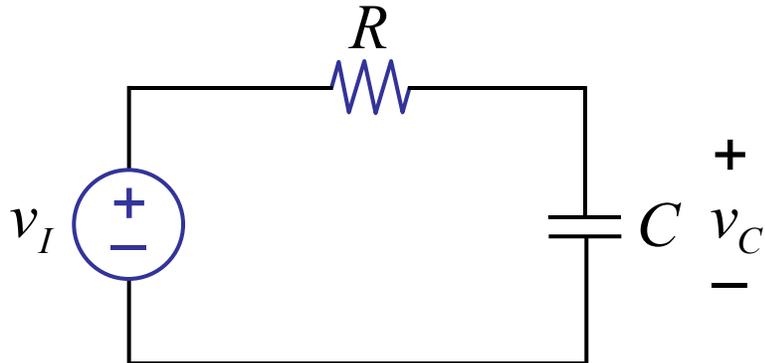


$$V_r = Z_r I_r$$

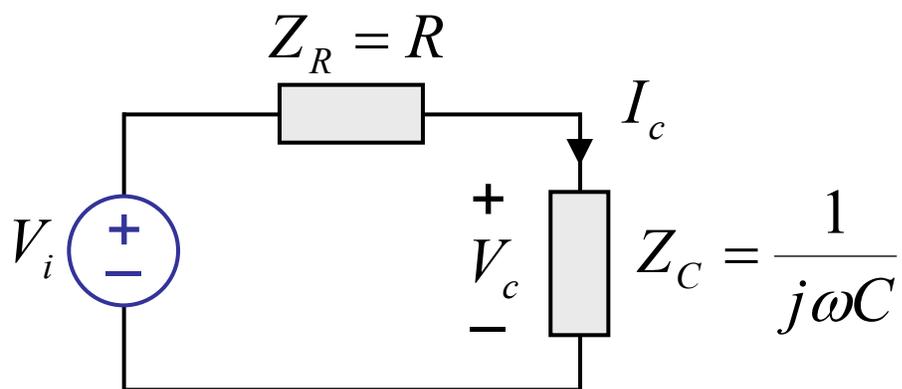
$$Z_r = R$$

For a drive of the form $V_c e^{j\omega t}$, complex amplitude V_c is related to the complex amplitude I_c algebraically, by a generalization of Ohm's Law.

Back to RC example...



Impedance model:



$$V_c = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} V_i = \frac{Z_C}{Z_C + Z_R} V_i$$

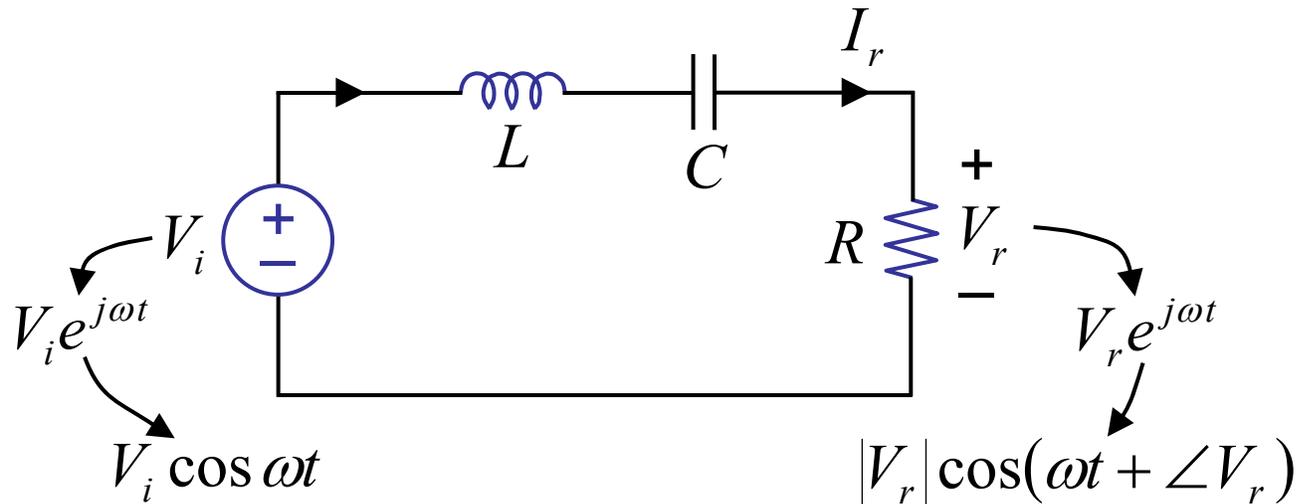
$$V_c = \frac{1}{1 + j\omega RC} V_i$$

Done!

All our old friends apply!
KVL, KCL, superposition...

Another example, recall series RLC:

Remember, we want only the steady-state response to sinusoid



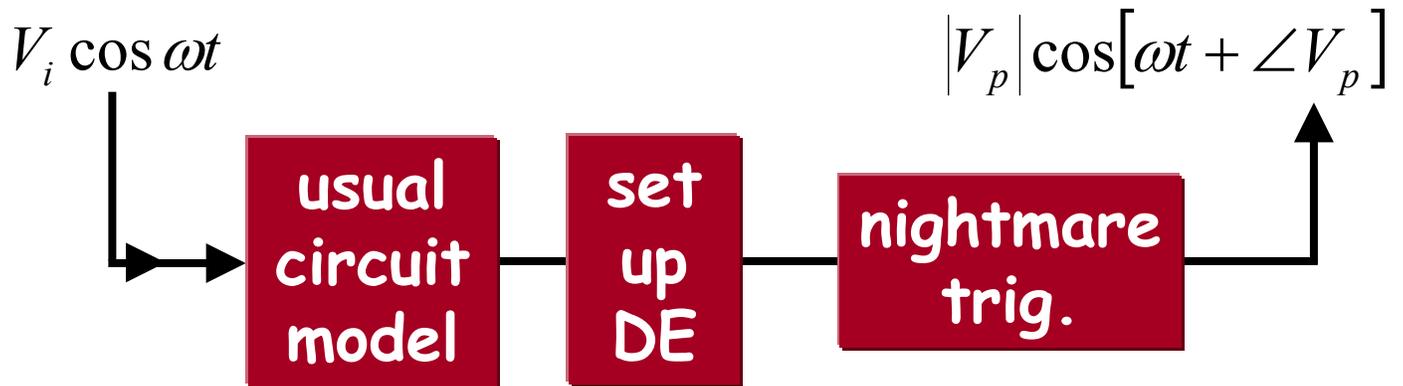
$$V_r = \frac{V_i Z_R}{Z_L + Z_C + Z_R}$$

$$V_r = \frac{V_i R}{j\omega L + \frac{1}{j\omega C} + R}$$

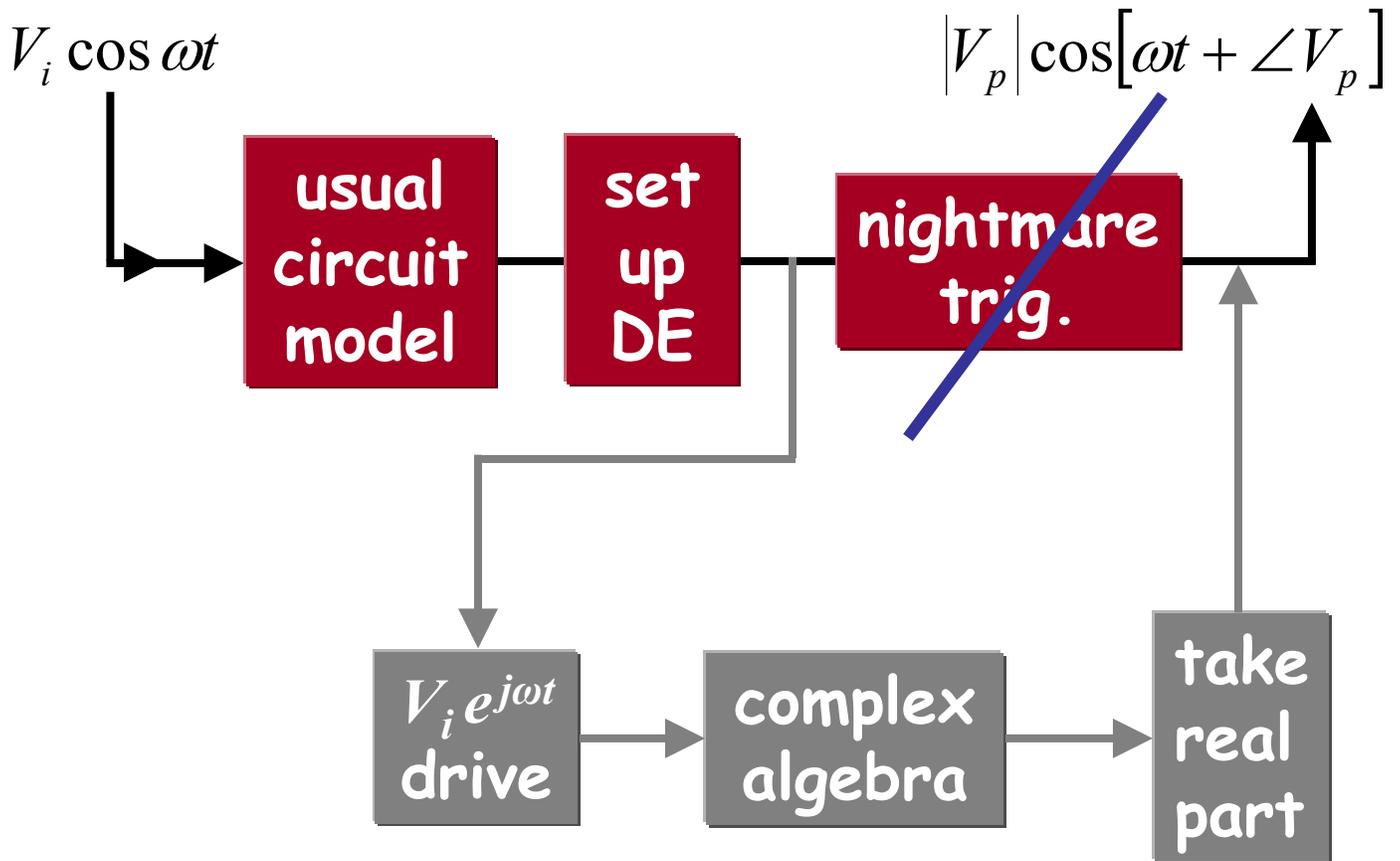
$$V_r = \frac{V_i j\omega CR}{-\omega^2 LC + 1 + j\omega CR}$$

We will study this and other functions in more detail in the next lecture.

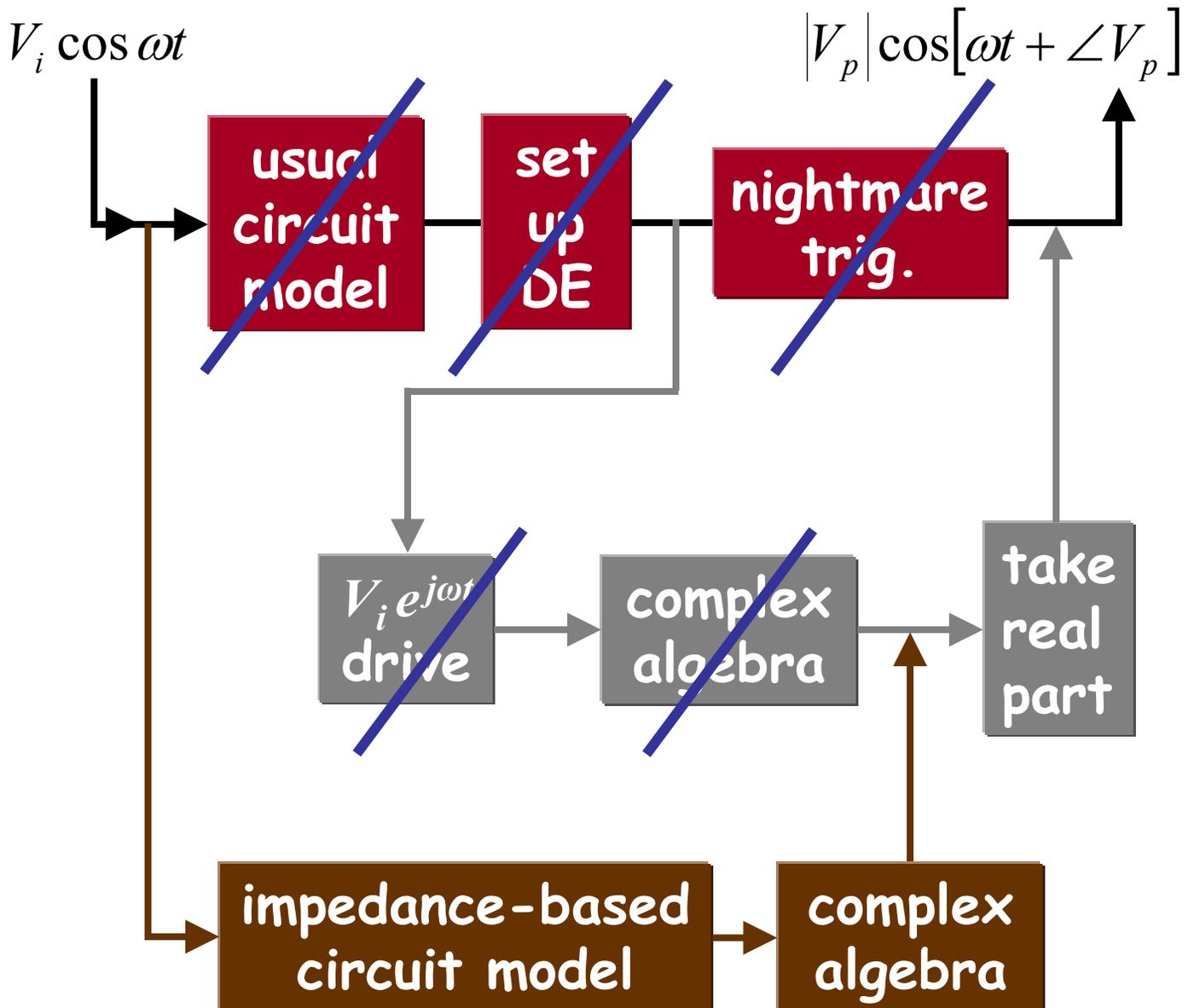
The Big Picture...



The Big Picture...



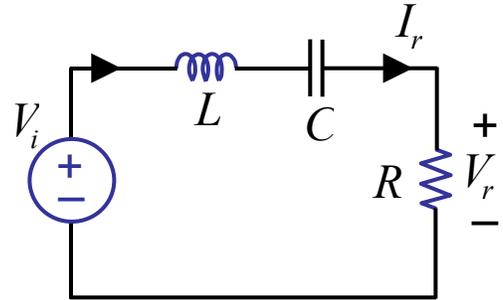
The Big Picture...



No D.E.s, no trig!

Back to

$$\frac{V_r}{V_i} = \frac{j\omega RC}{1 + j\omega RC - \omega^2 LC}$$



Let's study this transfer function

$$\frac{V_r}{V_i} = \frac{j\omega RC}{1 + j\omega RC - \omega^2 LC}$$

$$= \frac{j\omega RC}{(1 - \omega^2 LC) + j\omega RC} \cdot \frac{(1 - \omega^2 LC) - j\omega RC}{(1 - \omega^2 LC) - j\omega RC}$$

$$\left| \frac{V_r}{V_i} \right| = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$

Observe

$$\text{Low } \omega: \approx \omega RC$$

$$\text{High } \omega: \approx \frac{R}{\omega L}$$

$$\omega\sqrt{LC} = 1: \approx 1$$

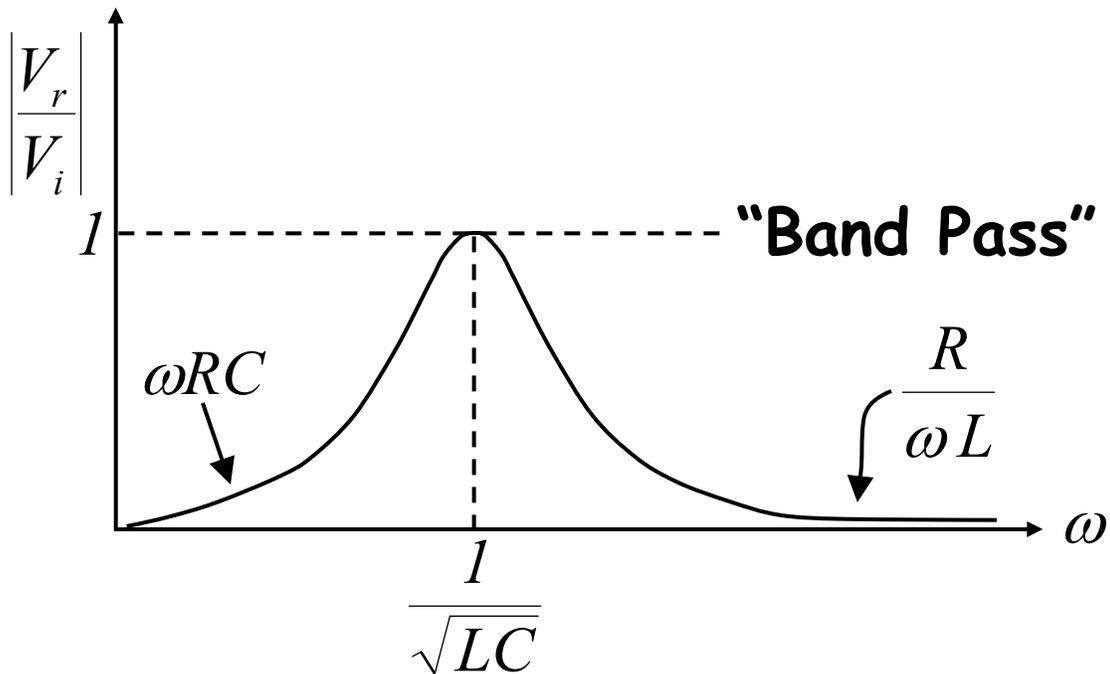
Graphically

$$\left| \frac{V_r}{V_i} \right| = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$

$$\text{Low } \omega: \approx \omega RC$$

$$\text{High } \omega: \approx \frac{R}{\omega L}$$

$$\omega\sqrt{LC} = 1: \approx 1$$



Remember this trick to sketch the form of transfer functions quickly.

More next week...